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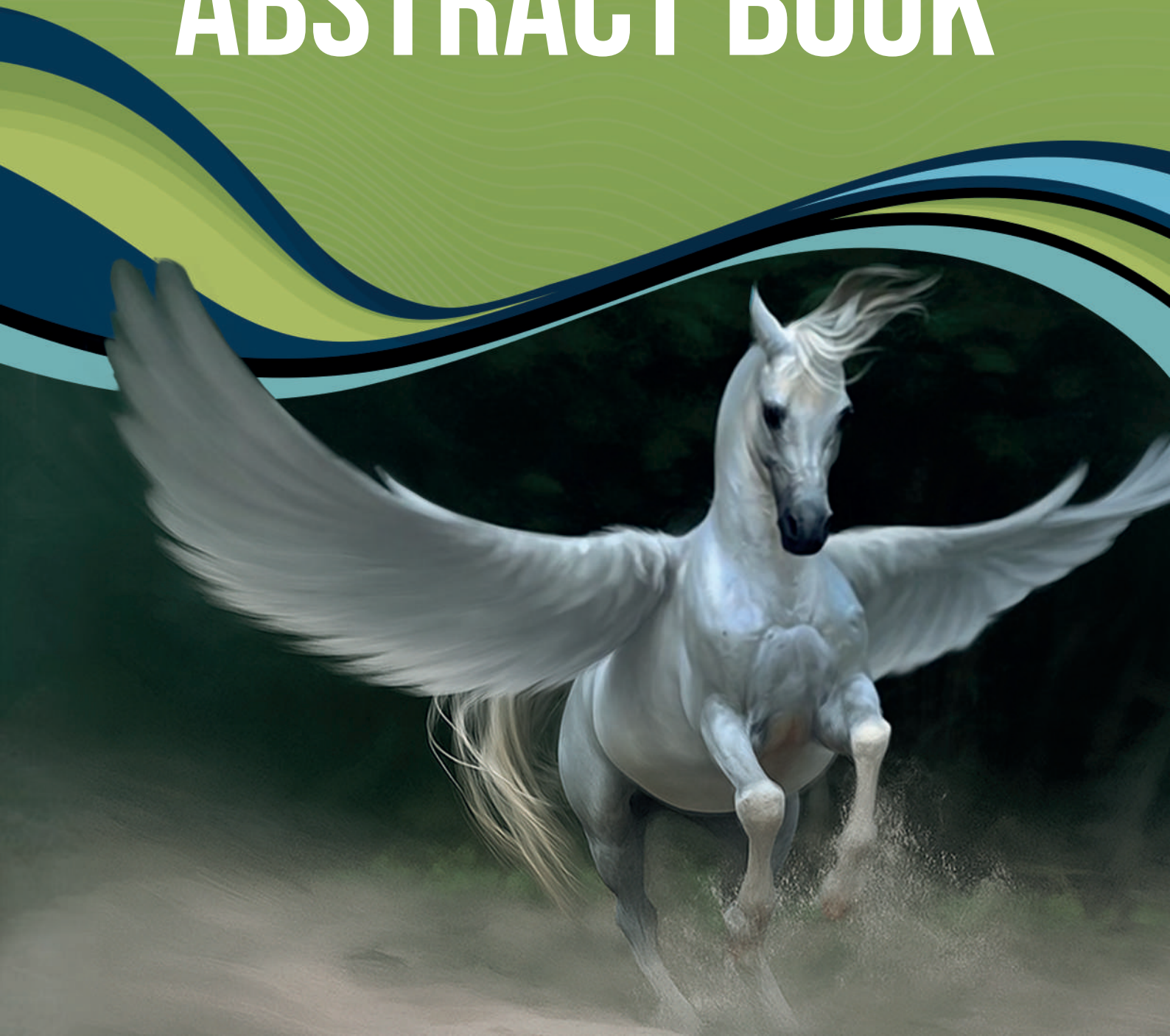
3RD INTERNATIONAL CONFERENCE: CONSTRUCTIVE MATHEMATICAL ANALYSIS

2-5 JULY 2025

ABSTRACT BOOK



The Third International Conference: Constructive Mathematical Analysis

Abstract Book

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FOREWORDS

Dear Participants,

It's my pleasure to chair The International Conference on "3rd International Conference: Constructive Mathematical Analysis". 3rd International Conference: Constructive Mathematical Analysis" is an activity of the journal Constructive Mathematical Analysis. The first two series of the event were organized as workshops. Based on the requests from international researchers studying on constructive mathematical analysis, this series was organized as a conference. Our 2025 conference was supported by Scientific Research Projects Coordinatorship of Selçuk University, The Scientific and Technological Research Council of Türkiye, Republic of Turkey Ministry of Youth and Sports Logo and BEYSU.

The purpose of this conference is to bring together experts and young analysts from all over the world working in different fields of mathematics and its applications to present their researches, exchange new ideas, discuss challenging issues, foster future collaborations and interact with each other.

The main goal of this conference is to promote, encourage, and provide a forum for the academic exchange of ideas and recent research works on any field of Analysis and Function Theory. The conference presented new results and future challenges, in a series of keynote lectures and contributed short talks.

We thank invited speakers distinguished **Prof. Francesco Altomare**, distinguished **Prof. Erdal Karapınar**, distinguished **Prof. Harun Karslı**, distinguished **Prof. Ioan Raşa**, distinguished **Prof. Gianluca Vinti**, distinguished **Prof. Xiaoming Wang** and distinguished **Prof. Ferenc Weisz** for contribution to the our conference.

The papers presented in this conference will be considered in the special issues of journals *ALTAY Conference Proceedings in Mathematics, Carpathian Mathematical Publications, Constructive Mathematical Analysis, Demonstratio Mathematica, Dolomites Research Notes on Approximation, Modern Mathematical Methods* and *Symmetry.*

This booklet contains the titles and abstracts of all invited and contributed talks at the *The Third International Conference: Constructive Mathematical Analysis* and is available on the conference website.

Prof. Dr. Tuncer ACAR
On Behalf of Organizing Committee - Chairman

to the memory of my uncle Hüseyin Acar

01.01.1966 - 02.01.2025

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Contents

3rd International Conference: Constructive Mathematical Analysis

Local Convergence Problems and Positive Approximation Processes on Function Spaces on Convex Sets and Convex Cones

Francesco Altomare

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key-words: Positive linear operator, local approximation, Bernstein-Chlodovsky operator.

Abstract:
Very recently, we developed some investigations ([1], [2], [3]) whose main aim was to deepen and to extend a general local convergence criterion for sequences of positive linear operators which was due to P. P. Korovkin ([6]).

The obtained new criteria guarantee whether a given linear function subspace, named here as local Korovkin subspace, satisfies the property that any sequence of positive linear operators locally converges on bounded locally continuous functions towards a given positive linear operator or towards the identity operator provided that the same property is checked on them.

The talk will be concerned with some new results in this direction which we have obtained in [1] and [2] where, among other things, it has been shown that for every given admissible subset S of continuous functions on a completely regular Hausdorff space, setting $S^2 := \{u^2 \mid u \in S\}$, then the linear subspace generated by $\{\mathbf{1}\} \cup S \cup S^2$ is a local Korovkin subspace for every weighted composition operator associated with a homeomorphic mapping and, in particular, for the identity operator.

As a consequence, we shall discuss some applications which concern the sequences of Bernstein-Schnabl operators acting on function spaces defined on convex Borel subsets as well as a new sequence of positive linear operators acting on function spaces defined on convex Borel cones.

Among other things such operators generalize a well-investigated modification of *Chlodovsky operators* on $[0, +\infty[$, also named *Bernstein-Chlodovsky operators*. Our approach describes a possibly unifying way on how to extend these operators to more general (both multidimensional and infinite-dimensional) settings as well.

The study is mainly addressed to highlight their pointwise convergence as well as their uniform convergence on compact subsets for functions which are bounded, Borel measurable and locally continuous.

In some particular cases, by means of such operators, a Weierstrass-type density result for the above mentioned class of functions in terms of polynomials is obtained as well.

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3rd International Conference: Constructive Mathematical Analysis

Primary Indispensable Remarks on Some Recent Results in Metric Fixed Point Theory

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key-words: Metric fixed point, contractions, abstract metric structures.

Abstract:
The aim of this talk is to reveal the dilemmas of the fixed point theorems that were started to be constructed by Banach in 1922. It is possible to transform many of the real-world problems into fixed point theorems. This shows how the fixed point theory is useful and how huge application potential it has. On the other hand, a lot of results are repeated in this theory, which is of great interest. A substantial part of the new results suggested overlaps with the old results. To support this observation, I shall share a few examples from the current literature.

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3rd International Conference: Constructive Mathematical Analysis

Wavelet-Based Approximation Operators: Applications to Bivariate Functions and Digital Image Processing

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key-words: Wavelet analysis, positive linear operators, approximation, bivariate operators, image reconstruction.

Abstract:
The aim of this talk is to demonstrate that the theory of wavelet-based operators plays a fundamental role in signal analysis and has significant applications in digital image processing. In classical Fourier analysis, signals are examined in the frequency domain by transforming them from the time domain. However, this transformation often results in the loss of time-related information, making the analysis of non-stationary signals particularly challenging. To address this limitation, various methods have been developed—one of the most prominent being wavelet analysis, originally introduced by Alfred Haar. Wavelets provide a powerful tool for decomposing signals into components that are localized in both time and frequency, allowing for more precise analysis of complex signals. This talk will focus especially on recent applications of wavelet methods to two-dimensional functions and image reconstruction. We will also explore how operators defined using specific types of wavelets serve as natural extensions of classical operators and their Kantorovich-type modifications. Notably, the wavelet-based operators and the associated algorithms presented here can be seamlessly applied to real-world problems traditionally handled by Kantorovich operators, thereby enhancing both the theoretical framework and practical effectiveness of signal and image processing techniques.
Acknowledgements: The author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The author thanks to TUBITAK for their supports.

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Tracial Functionals on Operator Algebras

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key-words: Trace, positive linear functional, matrix algebra, C^*-algebra, von Neumann algebra.

Abstract:
In this talk, we investigate tracial functionals on C^*-algebras and von Neumann algebras. We explore various characterizations of the standard trace on the full matrix algebra $\mathbb{M}_n$ and present several conditions under which states satisfy the tracial property. As applications, we establish new criteria for determining the commutativity of C^*-algebras.

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Sequences of Positive Linear Operators and Their Limits

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key-words: Positive linear operators, convergence, limit operator.

Abstract:
Usually, In Approximation Theory one investigates sequences of positive linear operators converging in some sense to the identity operator Id. However, some modifications of classical operators, inspired by the famous "Poisson approximation to the binomial distribution", lead to sequences of positive linear operators with limits different from Id. See, e.g., [2], [1] and the references therein. This talk is devoted to such sequences and to the characterization of their limits.

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Approximation Results for a General Family of Nonlinear Operators and Applications: A Unifying Approach

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key-words: Approximation, nonlinear operators, discrete operators, Orlicz spaces.

Abstract:
I will discuss some approximation results for a general family of nonlinear operators, proposing a general unifying approach. I will consider both pointwise and uniform convergence, and modular convergence in Orlicz spaces [4, 5]. Furthermore, I will also address the problem of qualitative estimates in suitable Lipschitz classes [6]. The theory presented allows to simultaneously treat families of discrete ([2, 1] and semi-discrete operators [3] (such as sampling or Durrmeyer-type sampling operators) and integral operators. The first ones (of sampling type) have also been studied in relation to concrete applications in the real world.

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Construction of Transition Layers Associated with a Few Fluid Models

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key-words: Transition layer, fluid model, stability, construction.

Abstract:
Most physical models contain parameters that are either very small or very large. The incompressible Navier-Stokes system at small viscosity (large Reynolds number) is a prime example. When the small parameter is set to zero, we often obtain a qualitatively different model whose behavior deviates significantly from that of the nonzero case. The zero-parameter model is usually simpler; therefore, we are interested in approximating the behavior of the nonzero model using the zero-parameter counterpart. However, such an approximation is not valid unless supported by a transition layer that bridges the two qualitatively different behaviors. The construction of such transition layers, as well as their stability and validity, will be the focus of this talk. Applications to the Navier-Stokes equations and the vanishing material interface limit of the convection in layered porous media, among others, will also be discussed. The results reported here are joint with H.J. Dong and R.M. Temam.

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Lebesgue Points and Summability of Higher Dimensional Fourier Series

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key-words: Higher dimensional Fourier series, Hardy-Littlewood maximal functions, Fejér means, Cesaro means, Lebesgue point.

Abstract:
We consider the convergence of partial sums and of summability methods for higher dimensional Fourier series. We focus on the Cesàro and Riesz methods and the ℓ_q- and rectangular summability. We study the norm and almost everywhere convergence. We investigate five different types of Hardy-Littlewood maximal operators and their weak and strong boundedness on L_p spaces. We characterize the set of the almost everywhere convergence as different types of Lebesgue points.

A restart derivative-free algorithm for solving pseudomonotone equations

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key-words: Iterative methods, Pseudo-monotone nonlinear equations, conjugate gradient method, Derivative-free method, Logistic regression.

Abstract: In this work, we propose an efficient derivative-free algorithm strengthened by a restart strategy for solving nonlinear pseudomonotone equations. We have shown that the algorithm's direction of search is descent and bounded and that the algorithm generates globally convergent sequences to the solutions under the assumptions of pseudomonotonicity and continuity. Moreover, the behavior of the convergent sequence near its limit is reveals for which we have shown that the sequence approaches its limit along the direction in the normal cone at the limit when the solution is not unique. Numerical simulations on benchmark test problems reveal the computational efficiency of our proposed algorithm in comparison with some existing recent ones. The suitability of the suggested algorithm for logistic regression problems is demonstrated, as well.

Neimark-Sacker Bifurcation and Control of Chaos in Prey-Predator System Subject to Harvesting Effect on Prey

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key-words: Prey-Predator system, Neimark-Sacker bifurcation, stability, bifurcation theory, harvesting effect, chaos control.

Abstract:
This article aims to examine the dynamics of the positive coexistence fixed point of the discrete-time model given in [3] by taking the conversion efficiency of prey into predator as the bifurcation point. We show that the system is exposed to Neimark-Sacker bifurcation. We also control the Neimark-Sacker bifurcation with a feedback control strategy. Additionally, computer simulations are presented to verify the theoretical results obtained with the stability, bifurcation and chaos control strategy.

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Integration of Modified Durrmeyer-Type Sampling Operators into Orlicz Spaces

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key-words: Durrmeyer type sampling operator, Orlicz space, modular convergence.

Abstract:
This document details the integration of modified Durrmeyer-type sampling operators into Orlicz spaces. The primary focus is on proving boundedness, establishing modular convergence, and deriving error estimates, ensuring that these operators can approximate functions within the Orlicz space framework effectively.
Acknowledgements: The author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The authors thank to TUBITAK for their supports.

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Generalized Split Equality Feasibility Problem in a Hadamard Space

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key-words: Mixed equilibrium problem, convex joint $\Delta-$lower semicontinuous function, generalized split equality feasibility problem.

Abstract:
In this paper, we introduce and study a Generalized Split Equality Feasibility Problem in the setting of Hadamard space. We construct a perturbed Halpern-viscosity type iterative algorithm and proved its strong convergence to solution of the above mentioned problem. Our technique of proof is new and of independent interest.

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Approximation with Nonlinear Lototsky-Durrmeyer Type Operators

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key-words: Lototsky operator, nonlinear Durrmeyer operator, rate of convergence.

Abstract:
In 1966, J. P. King [5] introduced Lototsky-Bernstein operators $L_n : C[0,1] \to C[0,1]$ for $n \in \mathbb{N}$ as follows:

$$L_n(f;x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right) a_{n,k}(x), x \in [0,1],$$

using the basis function $a_{n,k}(x)$ obtained from the following relation

$$\prod_{i=1}^{n} (h_i(x)\, y + 1 - h_i(x)) = \sum_{k=0}^{n} a_{n,k}(x)\, y^k, y \in \mathbb{R},$$

$$a_{n,k}(x) = \sum_{\substack{J \cup \bar{J} = \mathbb{N}_n \\ Card(J) = k}} \prod_{i \in \bar{J}} (1 - h_i(x)) \prod_{i \in J} h_i(x),$$

where $h_i : [0,1] \to [0,1]$ is a sequence of continuous functions and $a_{0,0}(x) = 1$, $a_{0,k}(x) = 0$ for $k > 0$. Recently, these operators have been studied by several mathematicians. Positive linear and nonlinear operators, convolution, moment and sampling operators have an important role in several branches of Mathematics. For instance, in reconstruction of signals and images, in Fourier analysis, operator theory, probability theory and approximation theory. In this way, the main objective of this work is to define a new sequence of nonlinear operators of type Lototsky-Durrmeyer and to give some convergence results for these type operators.

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On Some Families of Exponential Type Polynomials

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key-words: Exponential type polynomials, modulus of continuity, Korovkin theorems, constructive approximation, Lipschitz classes.

Abstract:
We will present approximation results by means of some exponential type polynomials that generalize, in different directions, the classical Bernstein polynomials.

In particular, we will introduce a multidimensional version of the Bernstein-type exponential polynomials on the hypercube $[0,1]^d$. For such operators, that fix product-type exponential functions and their square, we prove uniform convergence, by means of both the tools of Korovkin theory and a constructive approach, as well as a quantitative estimate of the order of approximation in terms of the modulus of continuity and, as a consequence, a qualitative result about the rate of convergence in some Lipschitz classes.

We will also discuss a new Kantorovich version of the exponential polynomials for which uniform convergence, convergence in weighted L^p-spaces, a Voronovskaja formula and results about the rate of approximation are established.

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Recent Results on the Weighted Approximation

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key-words: Weighted approximation, weighted modulus of continuity, Korovkin and quantitative type theorems.

Abstract:
In this presentation, we express Korovkin and quantitative theorems under the weighted approximation. After giving the basic definitions and some of well-known spaces, we give the main theorems and their applications to linear positive operators. Moreover, we state a general operator containing well-known operators as an application of some theorems.
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Approximation Results on Neural Network Operators of Convolution Type

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key-words: B-generalized logistic function, convolution type, rate of convergence, iterated approximation.

Abstract:
In the present paper, we introduce three neural network operators of convolution type activated by symmetrized, deformed and parametrized B-generalized logistic function. We deal with the approximation properties of these operators to the identity by using modulus of continuity. Furthermore, we show that our operators preserve global smoothness and consider the iterated versions of them. Here, we find it is worthy to mention that these operators play important roles in neural network approximation since most of the basic network models are activated by logistic functions.

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3rd International Conference: Constructive Mathematical Analysis

On Uniform Exhaustiveness and Korovkin-Type Theorem

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key-words: Function sequences, exhaustiveness, Korovkin-type theorems.

Abstract:
This study considers a uniform version of exhaustiveness for function sequences and examined its connections with alpha convergence, uniform alpha convergence, and classical uniform convergence. A Korovkin-type approximation theorem is also established, showing that the classical conditions can be loosened under this new concept.

Let (X, d) and (Y, ρ) be metric spaces, (f_n) be a sequence of functions and f be a function from X to Y.
Definition 1. [2] The function sequence (f_n) is called *exhausitive* at $x_0 \in X$, if for every $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon, x_0) > 0$ and $n_0 = n_0(\varepsilon, x_0) \in \mathbb{N}$ such that for all $x \in B_d(x_0, \delta)$ and all $n \geq n_0$ we have that $\rho(f_n(x), f_n(x_0)) < \varepsilon$.
Definition 2. [3] The function sequence (f_n) is called *uniform exhausitive* on X if for every $\varepsilon > 0$ there exists $n_0 = n_0(\varepsilon) \in \mathbb{N}$ and $\delta = \delta(\varepsilon) > 0$ such that for all $n \geq n_0$ and for all point $x, y \in X$ with $d(x, y) < \delta$ we have that $\rho(f_m(x), f_m(y)) < \varepsilon$.
Definition 3. [1] The function sequence (f_n) is called *uniform alpha convergent* to the function f on X if for every sequence (x_n) and (y_n) of points of X with the property $d(x_n, y_n) \to 0$, the sequence $\rho(f_n(x_n), f(y_n)) \to 0$.

Relationships with previously defined notions and classical uniform convergence are examined. Furthermore, a Korovkin-type approximation theorem is given within this new notion of exhaustiveness, emphasizing its significance in approximation theory. It is observed that the conditions in the classical Korovkin theorem can be loosened under this new notion.

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A Note on Modified Landau Type Operators

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key-words: Modulus of continuity, Landau operator, weighted approximation.

Abstract:
In this study, we define modified Landau type operators. We investigate several convergence results concerning these operators. Specifically a quantitative approximation theorem and Voronovskaya type theorem are given using the modulus of continuity. Similar conclusions are also given in weighted spaces.

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On Some Direct Estimates Results of the Rate Approximation by the Kantorovich Operator in Weighted Grand Lebesgue Spaces

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key-words: Weighted grand Lebesgue spaces, Hardy-Littlewood maximal operator, Kantorovich operator.

Abstract:
In this study, we prove the boundedness of the Kantorovich operator K_n in weighted grand Lebesgue spaces $L_{\omega}^{p),\delta}(\Omega)$, where $\Omega \subset R^N$ is a open bounded subset. Moreover, we establish two direct estimates by K-functionals of the rate of approximation in $L_{\omega}^{p),\delta}(\Omega)$. Finally, we extend the direct estimate inequality in classical Lebesgue spaces $L_p[0,1]$ to $L_{\omega}^{p),\delta}$.

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On Some Properties and Compactness Criterions of Variable Exponent Fofana's Space

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key-words: Amalgam spaces, Hardy-Littlewood maximal operator, variable exponent Fofana's spaces.

Abstract:
In functional analysis, the Kolmogorov-Riesz (or Fréchet–Kolmogorov) theorem gives a necessary and sufficient condition for a set of functions to be relatively compact in an $L^p[0,1]$ space $(1 < p < \infty)$ by Kolmogorov [4]. Compactness results in the spaces $L^p\left(\mathbb{R}^d\right)$ are quite important in existence of solutions and proofs for nonlinear partial differential equations. This theorem has been generalized to some function spaces, such as Tamarkin [7], Olsen and Holden[3], Rafeiro [6], Gorka and Macios [2], Pandey [5], Aydın and Unal [1]. In this regard, we recommend some new theorems for compactness criterions of bounded subsets in Varaible Exponent Fofana's Spaces. Finally, we obtain also a generalization of the corresponding aboved studies.

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Calculation of Two-Center Coulomb Integrals over STOs in Nonlined-up Coordinate System

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key-words: Slater-type orbitals, Coulomb integrals.

Abstract:
One of the most commonly used approaches in electronic structure calculations of many-electron systems is the Hartree-Fock-Roothaan (HFR) approach. According to this approach, the HFR equation must be solved to obtain the wave function (Slater determinant) of the molecule. One of the terms of the HFR equation is the Fock matrix, one is the kinetic energy of the electron and the interaction potential energy of the electron with the nucleus, and the other is the electron repulsion potential energy term resulting from the interaction of the electron with other electrons. The most important of the electron repulsion potential energy is the Coulomb potential energy term. In addition to these, the Coulomb potential energy must be calculated accurately in the solution of the Dirac equation (Bagci and Hoggan, 2015).

As can be understood from what we have said, it is very important to calculate the Coulomb potential energy accurately in order to obtain the wave function (Slater Determinant) of the quantum system using the HFR approach. In addition, in the calculation of the orbital energies of atoms or molecules, the most important term after the kinetic energy of the electron and the interaction potential energy with the nucleus is again the Coulomb potential energy term between electrons. In addition to what has been said, the Coulomb potential energy must also be calculated sensitively in the solution of the Dirac equation.

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Application of Wavelet Transformation in Analyzing Railway Track Profile Irregularities

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key-words: Railway track analysis, wavelet transformation, rail profile irregularities.

Abstract:
Wavelet transformation provides a time-frequency analysis of non-stationary signals. It enables the detection of local events occurring within short time intervals. This capability is particularly critical for the analysis of railway track profile geometry [1], which often exhibits rapid and abrupt vertical irregularities over specific distances due to various mechanical factors. These factors include local voids in the supporting soil, rapid variations in track layer thickness, ballast wear and contamination, existence of rail-ends and turnouts, and the discontinuous support provided by sleepers. Inspection vehicles equipped with special measurement sensors can convert these geometric changes into a continuous signal.

Through multi-level wavelet decomposition, the dominant characteristics of the track profile signal can be extracted, along with detailed sub-signals of different frequencies and wavelengths [2]. These sub-signals represent the contribution of each underlying cause to the observed rail profile irregularities [3]. This study presents a case study where wavelet decomposition is applied to a track profile data collected from UK. The track profile is isolated into sub-parts and possible causes of the overall deformation is investigated independently. The findings can contribute to the developing effective maintenance strategies that enhance safety and extend the service life of railway tracks.

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Spectral Analysis of The Impulsive Sturm-Liouville Equation with Quadratic Spectral Parameters and Matrix Coefficients

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key-words: Scattering matrix, impulsive condition, eigenvalues, spectral singularities.

Abstract:
The aim of this study is to investigate some spectral and scattering properties of an impulsive Sturm-Liouville equation with matrix coefficient dependent on the spectral parameter in quadratic form. Firstly, we obtain the Jost solution and the Jost matrix. Then, we find the sets of eigenvalues and spectral singularities of our problem by using the resolvent operator and the Green matrix. Furthermore, we demonstrate that the impulsive differential equation has finite number of eigenvalues and spectral singularities with finite multiplicities under certain conditions. We also obtain the scattering matrix of our problem and examine the properties of the scattering matrix.

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Quantitative Estimates for Wavelet Type Extension of Generalized Kantorovich Operators

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key-words: Generalized Kantorovich operators, wavelets, K-functional of Peetre, Bernstein-Markov type inequalities.

Abstract:
We consider a sequence of wavelet type modifications of univariate generalized Kantorovich operators depending on a positive real parameter given in [2].
In our talk; we present quantitative estimates for the rate of convergence of the operators in the spaces of continuous and pth power integrable functions defined on a suitable compact interval. The related estimates in both cases are expressed in terms of the modulus of continuity and the Peetre K-functional of the approximated function. Furthermore, we provide some inequalities such as Bernstein-Markov type for continuous functions and variation preservation property of the operators when the involved function is of bounded variation.

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Robust Estimation of Parameters in Simple Linear Regression Model

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key-words: Linear regression model, robust estimation methods, outliers, Monte-Carlo simulation.

Abstract:
This study investigates parameter estimation within a simple linear regression model, assuming that the error terms have a non-normal distribution. Robust estimation techniques, such as least absolute deviation, weighted least absolute deviation, and least median of squares, were employed to estimate the unknown model parameters. The performance of these estimators was evaluated using various comparison criteria. A Monte-Carlo simulation study was conducted on data sets both with and without outliers (in the y-direction). The results reveal that the least median of squares estimator outperforms the others in numerous scenarios, irrespective of the presence of outliers. Furthermore, the effectiveness of the proposed estimators is illustrated through an analysis of a data-set, which was intentionally altered with varying percentages of outliers.

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Minimum Hellinger Distance Estimate for a Periodic Fractional Autoregressive Model of First-Order

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key-words: Hellinger distance, estimation, peridic fractional autoregressive model.

Abstract:
In this paper, we develop an estimator for the parameter of a first-order periodic fractional autoregressive model following the minimum Hellinger distance approach. Then, we investigate two of its key properties, which are its consistency and limit distribution. A simulation study will demonstrate the most important findings of the study.
Model and notations:
A stochastic process $\{X_t, t \in \mathbb{Z}\}$ is said to be the fractional process of first-order $FAR(1)$ with a periodic coefficient denoted $PFAR_{a_p}$ if it has the following representation

$$(1 - a_t L)^d X_t = \varepsilon_t \tag{1}$$

- $\theta_t = (a_t; d) \in \mathbb{R}^2$ is an unknown vector, such that for all $t \in \mathbb{Z}, \exists i = \{1, ..., p\}$ and $M \in \mathbb{Z}$ such that $t = i + pM$ and so $a_t = a_{i+pM} = a_i$, here $p \geqslant 2$, we also suppose that $a_t.d \neq 0$.

The minimum Hellinger distance estimator

In this paper, we will define an estimator of θ and let it be denoted by $\hat{\theta}_{M,n}$. This estimator minimizes the Hellinger distance (HD^2) between the theoretical probability density of η_M and a random probability density of the estimated η_M (referred to as $\hat{\eta}_M$) denoted by $f_\eta(.)$ and $f_n(.)$ respectively; $\hat{\theta}_{M,n}$ is hence named the **M**inimum **H**ellinger **D**istance (MHD) estimator, and it is defined in the following form

$$\hat{\theta}_{M,n} = arg \min_{\theta \in \Theta} HD^2(f_n; f_\eta) \tag{2}$$

where $HD^2(f_n; f_\eta)$ is the L_2-norm of the deviation between the square-roots of f_n and f_η.

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Solvability of a System Arising in Nonlinear Ultrasonic Heating

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key-words: JMGT equation, nonlinear acoustics, Pennes bioheat equation.

Abstract:
We analyze a mathematical model of nonlinear ultrasonic heating based on the Jordan-Moore-Gibson-Thompson equation (JMGT) with temperature-dependent medium parameters coupled to the semilinear Pennes equation for the bioheat transfer. Precisely, we consider the model

$$\begin{cases} \tau p_{ttt} + p_{tt} - c^2(\bar{\theta})\Delta p - \delta(\bar{\theta})\Delta p_t = K(\bar{\theta})\left(p^2\right)_{tt}, & \text{in } \Omega \times (0,T), \\ \rho_a C_a \bar{\theta}_t - \kappa_a \Delta \bar{\theta} + \rho_b C_b W(\bar{\theta} - \theta_a) = \mathcal{Q}(\bar{\theta}, p_t), & \text{in } \Omega \times (0,T), \\ (p, p_t, p_{tt}, \bar{\theta})|_{t=0} = (p_0, p_1, p_2, \bar{\theta}_0), & \text{in } \Omega, \\ p|_{\partial\Omega} = 0, \qquad \bar{\theta}|_{\partial\Omega} = \theta_a, \end{cases} \tag{1}$$

where Ω is a bounded domain of $\mathbb{R}^d$, $d = 1, 2, 3$. The functions p and $\bar{\theta}$ denote the acoustic pressure and the temperature fluctuations, respectively, $c(\bar{\theta})$ is the speed of sound and $\delta(\bar{\theta})$ is the sound diffusivity. The parameter $\tau > 0$ denotes a positive constant accounting for the relaxation time and the function $K(\bar{\theta})$ is also allowed to depend on $\bar{\theta}$ and is given by $K(\bar{\theta}) = \dfrac{\beta_{\text{acous}}}{\rho c^2(\bar{\theta})}$, where ρ is the mass density and β_{acous} is the parameter of nonlinearity. As for the parameters in the Pennes equation, ρ_b, C_b are the density and specific heat capacity of blood, and W is the tissue's volumetric perfusion. We denote by ρ_a, C_a and κ_a, respectively, the ambient density, the ambient heat capacity and the thermal conductivity of the tissue. Moreover, θ_a stands for the ambient temperature and $\mathcal{Q}$ for the acoustic energy absorbed by the tissue.

Given the quasi-linearity of the pressure equation, we use a higher-order energy method together with a fixed-point argument to prove the local well-posedness of our model. This is achieved under the assumptions that the initial data are regular, small in a lower topology and that the final time is sufficiently short. In addition, making use of the uniformity of the derived estimates with respect to the time relaxation parameter τ, we determine the convergence rate of the solution of the JMGT–Pennes model to the solution of the Westervelt–Pennes model as $\tau \to 0$. For more details, see [1].

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On Modular Equation and Related Distortion Functions

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key-words: Special functions, metrics, elliptic integrals.

Abstract:
In this paper, we study distortion functions derived from the solutions of modular equation arising in number theory. We establish their monotonicity and convexity properties, and derive sharp bounds. Moreover, we discuss the role of distortion functions in the theory of quasiconformal mappings.

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Estimation of a Finite Population Mean Under Random Non-Response Using Improved Nadaraya-Watson Kernel Weights

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key-words: Mean squared error, bias, kernel regression, two-stage cluster sampling, confidence interval lengths.

Abstract:
Non-response is a potential source of errors in sample surveys. It introduces bias and large variance in the estimation of finite population parameters. Regression models have been recognized as one of the techniques of reducing bias and variance due to random non-response using auxiliary data. In this study, it is assumed that random non-response occurs in the survey variable in the second stage of cluster sampling assuming full auxiliary information is available throughout. Auxiliary information is used at the estimation stage via a regression model to address the problem of random non-response. In particular, auxiliary information is used via an improved Nadaraya-Watson kernel regression technique to compensate for random non-response. The asymptotic bias and mean squared error of the estimator proposed are derived. Besides, a simulation study conducted indicates that the proposed estimator has smaller values of the bias and smaller mean squared error values compared to existing estimators of finite population mean. The proposed estimator is also shown to have tighter confidence interval lengths at 95% coverage rate. The results obtained in this study are useful for instance in choosing efficient estimators of finite population mean in demographic sample surveys.

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Spectrum of the Operator $H(r,s,t,u,v)$ Defined on the Sequence Spaces ℓ_p and bv_p

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key-words: Spectrum of an operator, resolvent set, perturbed operator.

Abstract:
In this work, we determine the fine spectrum of quintuple band matrix operator $H(r,s,t,u,v)$ which is defined on the sequence spaces ℓ_p and bv_p, where $1<p<\infty$. The quintuple band matrix $H(r,s,t,u,v)$ generalizes the $D(r,0,s,0,t)$, Δ^4, $Q(r,s,t,u)$, Δ^3, $D(r,0,0,s)$, $B(r,s,t)$, Δ^2, $B(r,s)$, Δ, right shift and Zweier matrices.

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Recent Results on Sampling Kantorovich Operators of the Max-Product Type in Functional Spaces

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key-words: Max-product sampling Kantorovich operators, Orlicz spaces.

Abstract:
In recent years, the max-product approach, originally introduced by Bede, Coroianu and Gal (for a complete overview, see the monograph [2]), has been applied to several families of linear operators. The main advantage of max-product type operators–constructed using the maximum (or supremum in the case of infinite terms) instead of the usual sum (or series for infinite terms)–is that they generally offer sharper approximations, in terms of better orders of convergence, than their linear counterparts. Motivated by these considerations, in [5], the authors introduced and studied the max-product version of the Kantorovich sampling series [1] based on generalized kernels, obtaining convergence results in the space of uniformly continuous and bounded (non-negative) functions, as well as in the L^p-spaces, for $1 \leq p < +\infty$.
In the present talk, we will show a recent study on the approximation properties of the above non-linear (sub-additive) sampling operators in the more general setting of Orlicz spaces L^{φ}, generated by convex φ-functions. In recently published papers [3, 4], we investigated the problem of convergence, including the rate of approximation, for sequences of max-product sampling Kantorovich operators with respect to the general notion of modular convergence, typical of such functional spaces. In addition to obtaining a convergence result via a density approach, we also established a quantitative estimate for the order of approximation in the case of functions $f : \mathbb{R} \to \mathbb{R}_0^+$, $f \in L^{\varphi}(\mathbb{R})$, by exploiting the Orlicz-type modulus of smoothness $\omega(f, \delta)_{\varphi}$, with $\delta > 0$, defined in terms of the modular functional of the corresponding space. We will show that from this result it is possible to deduce, for non-negative functions belonging to suitable Lipschitz classes in Orlicz spaces, a qualitative estimate proving that the rate of convergence is affected by the regularity of both the kernel function and the function to be approximated.
Since, as it is well-known from the literature (see, e.g., [6]), these general spaces include a wide class of functional spaces–such as Lebesgue spaces, among others– the treatment in this general context offers the advantage of formulating a unifying theory for the convergence properties of the above operators.

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Approximation Properties of a New Generalization of Brass-Stancu Operators in L^p-Spaces

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key-words: Brass-Stancu-Kantorovich-type operators, L^p-convergence, averaged modulus of smoothness.

Abstract:
In this presentation, we consider the operators defined by Brass [3] and Stancu [4]. Using the Kantorovich operators in [1], we obtain another Kantorovich-type modification of these operators as in [2]. We investigate the L^p-approximation properties of these new generalized operators. Specifically, the present study focuses on L^p-convergence and provides some upper estimates for the L^p-norm of the approximation error via first-order averaged modulus of smoothness and the first-order K-functional. In addition, we provide some numerical and graphical examples to demonstrate the rate of convergence of the constructed operators.
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Cross–Approximation of Functions of 2 Variables

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key-words: Approximation, bivariate function, cross-function.

Abstract:
Let $f(x, y)$ be a function defined on $[0, 1]^2$. The functions $f_{ab}(x, y) = f(a, y)f(x, b)$ $(a, b \in [0, 1])$ are called crosses generated by f. S. Mazur (1936) asked whether any continuous function can be uniformly approximated by linear combinations of its crosses with any accuracy. A. Grothendieck (1955) showed that this problem is in a certain way equivalent to the famous approximation problem. Since P. Enflo (1973) solved the approximation problem negatively, the answer to Mazur's question is 'no', and A. Davie (1975) constructed the corresponding example explicitly.

We present several conditions on a continuous function f that are sufficient for linear combinations of its crosses to approximate it with any accuracy.

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Fixed Point Results for Generalized Cyclic $\alpha - f\pi\varpi-$Contractions

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key-words: b-metric-like space, fixed point, cyclic contractions.

Abstract:
In this work, we introduce generalized cyclic contractions in the context of metric-like space and prove some new fixed point results concerning these contractions. We present suitable examples to make our findings worth mentioning.

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Compactness Result in the Set of Measures

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key-words: σ-algebras, measure, duality, compactness, weak topology, uniform integrability.

Abstract:
We consider a σ-algebra $\mathcal{B}$ on a space X and we denote by $\underline{\mathcal{B}}$ (resp. $\mathcal{M}$) the set of all real bounded and $\mathcal{B}$-measurable functions on X (resp. the set of all finite real measures on $\mathcal{B}$).
On the real vector space $\underline{\mathcal{B}} \times \mathcal{M}$ we consider the natural duality

$$(f, \mu) \mapsto \int f d\mu. \tag{1}$$

We give a complete characterisation of the compact subsets of $\mathcal{M}$ with respect to the weak topology on $\mathcal{M}$ associated with the above duality.
We remember that the Dunford-Pettis theorem gives a partial answer namely: if μ_0 is a positive real measure and K is a subset of $L^1(\mu_0)$, then K is relatively compact with respect to the duality $\left(L^1(\mu_0), L^\infty(\mu_0)\right)$ if K is uniformly integrable with respect to μ_0.

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A Converse of Triangle Inequality: Applications

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key-words: Abel convergence, Banach spaces, triangle inequality, radius of convergence, power series.

Abstract:
In any metric space (M, d), giving three points x_1, x_2, x_3 such that

$$d(x_1, x_2) < d(x_1, x_3) \tag{1}$$

we have

$$1 \leq \frac{d(x_2, x_3)}{d(x_1, x_3) - d(x_1, x_2)}. \tag{2}$$

In some cases we have

$$\sup \left\{ \frac{d(x_2, x_3)}{d(x_1, x_3) - d(x_1, x_2)} \middle| x_2 \in M \right\} < \infty, \tag{3}$$

for example in Hilbert (resp. Banach) spaces we have

$$\frac{d(x, x_3)}{d(x_1, x_3) - d(x_1, x)} < \frac{d(x_2, x_3)}{d(x_1, x_3) - d(x_1, x_2)}, \forall x \in co(x_1, x_2, x_3). \tag{4}$$

$$\text{(resp. } \frac{d(x, x_3)}{d(x_1, x_3) - d(x_1, x)} < \frac{\max(d(x_2, x_3), d(x_1, x_3))}{d(x_1, x_3) - d(x_1, x_2)}. \tag{5}$$

An open question arises here.
We use this result to study the behaviour of a holomorphic function at the boundary.

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On Von Neumann Approximation Theorem

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key-words: Weierstrass density theorem, Bernoulli inequality, Uryson family, Von Neumann density.

Abstract:
In one of his article on logics Von Neumann stated, without proof, that a set $\mathcal{V}$ of continuous functions on $[0,1]^n$ with values in $[0,1]$, uniformly approximates any $[0,1]$-continuous function on $[0,1]^n$ wherever $\mathcal{V}$ is closed to the multiplication operation, contains the projection functions and

$$1-f\in\mathcal{V}\quad\forall f\in\mathcal{V}. \tag{1}$$

We develop some technics which may be useful not only to prove the Neumann result.
We improve also the methods used by Jewett in this area.

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Some Novel Fixed Point Results in the Sense of Interpolative Metric Space

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key-words: Fixed point, interpolative metric space, simulation functions, integral equations.

Abstract:
In this study, we investigate a novel fixed point theorem within the framework of interpolative metric spaces, an enriched generalization of classical metric structures. By extending the scope of contractive-type conditions in this broader setting, we unveil new existence and uniqueness results that contribute to the ongoing development of nonlinear analysis. The proposed approach not only refines previous fixed point results but also offers greater flexibility in handling mappings that elude standard metric assumptions. Furthermore, we present a pioneering application of our main theorem to a nonlinear integral equation, demonstrating both the theoretical depth and the practical relevance of our findings. This work opens a promising avenue for future research, particularly in areas where classical fixed point results fall short due to structural constraints of the underlying space.

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Convergence in the Variation Seminorm by Generalized Kantorovich-Type Szász-Mirakyan Operators Constructed via Appell Polynomials

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key-words: Linear positive operators, generalized Szász-Mirakyan operators, variation seminorm, variation detracting property, rate of convergence, convergence in variation seminorm, TV [I]-space, Appell polynomials.

Abstract:
The aim of this study is to investigate the variation detracting property and convergence in variation of the generalized Kantorovich type Szász mirakyan operators constructed via Appell polynomials in the space of functions of bounded variation. These problems are examined with respect to the variation seminorm. Additionally, the rate of convergence is analyzed in terms of total variation.
Acknowledgements: The first author is supported by Scientific and Technological Research Council of Turkey (TUBITAK) for the graduate scholarship.

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A Novel Inflated Mixture Distribution with Exponential and Unit Lindley Components

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key-words: Inflated model, mixture distribution, bootstrap, point estimation.

Abstract:
In practical applications, there is an increasing need for more flexible distribution models to capture diverse data characteristics better. In this study, a new flexible mixture distribution is proposed. The components of this mixture are the exponential distribution and the unit-Lindley distribution [1] transformed to the (a, b) interval. This new mixture distribution differs from existing mixture distributions in the literature. Specifically, it possesses a mixture structure that inflates the (a, b) interval. Some fundamental statistical properties of the proposed distribution are examined, and graphical presentations for the probability density function and hazard rate function for various parameter settings are also provided. The maximum likelihood estimation method is used for the point estimation of the unknown parameters, and confidence intervals for the parameters are obtained based on the asymptotic properties of the maximum likelihood estimation as well as the bootstrap approach. The performances of both the point and interval estimators are evaluated through a Monte Carlo simulation study. A real data analysis is conducted to demonstrate the practical applicability of the proposed model.

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Disjoint $\mathcal{F}$-transitive Composition Operators on Orlicz Spaces

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key-words: Disjoint $\mathcal{F}$-transitivity, composition operator, Orlicz space.

Abstract:
In this talk, we will give sufficient and necessary conditions for some families of composition operators on the Orlicz space to be disjoint $\mathcal{F}$-transitive, where $\mathcal{F}$ is a Furstenberg family of a set. By considering some specific Furstenberg families of the set of integers, some previous results are recovered. This is a joint work [1] with Prof. Bagheri Salec, Prof. Tabatabaie and Prof. Al-Shammari.

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Best Proximity Points for Cyclic Operators with Applications in Image Processing

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key-words: Best proximity point, cyclic operator, image processing.

Abstract:
Our aim is to present recent results regarding best proximity point theory with respect to cyclic operators. Inspired by the a cyclic operator and we propose a new algorithm which gives a technique to stitch a pattern.

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Completeness Theorems on the Boundary

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key-words: Completeness theorems, elliptic equations of higher order, elasticity system, parabolic equations.

Abstract:
Roughly speaking, there are two different kinds of completeness theorems for systems of particular solutions of partial differential equations. Results of the first kind show that in a given norm we can approximate a solution of a partial differential equation in a domain by a sequence of particular solutions of the same equation. For example, if we have a holomorphic function f of one complex variable, we can ask when f can be approximated in some norms by polynomials or by rational functions. The classical theorems of Runge and Mergelyan are the main results in this direction. These problems have been widely studied and extended to general elliptic partial differential equations.

Results of a second kind are the so-called completeness theorems in the sense of Picone. They are much more sophisticated and refer not only to a PDE, but also to a certain BVP. Given a BVP in a domain Ω, the problem is to prove that the system obtained by applying the boundary operator of the BVP to particular solutions of the PDE is complete in an appropriate space on $\partial\Omega$.

The aim of the present talk is to discuss this second topic. In particular, we will focus on the completeness on the boundary of systems obtained by means of polynomial solutions. After a review of the previous results, we present the most recent ones. They concern elasticity system [1], higher order scalar elliptic equations in two variables [2] and parabolic equations [3].

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On Three-Layer Conjugation Problem for the Heat Conduction Equation in a Rectangle

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key-words: Two-layer medium, temperature field, unsteady heat conduction, method of integral transformations, conjugate conditions.

Abstract:
The analytical theory of unsteady heat conduction in a rectangular three-layer plate finds wide application in solving important technical and technological problems expressing Fourier's and Fick's laws under the corresponding dependence of the internal energy of the medium on temperature. In the article the initial boundary value problem for the heat conduction equation in a rectangular plate with a given initial temperature distribution, boundary conditions and conjugation conditions on the rupture line of the rectangular plate are considered. Thermodynamic processes are accompanied by heat transfer between elements of structures in the working space of thermal plants and the surrounding space, in which heat is a quantitative measure of energy and between bodies there is heat exchange and regularities of the process of heat transfer are considered by the theory of heat exchange and heat transfer.The analytical solution of the problem of unsteady heat conduction in a three-layer medium is constructed by the method of integral transformations: Laplace on the time variable and by the finite Fourier transform on the coordinate variables. The study of initial boundary value problems for the heat conduction equation and the creation of analytical methods for solving such problems represent an important and rapidly developing area of mathematical physics The solution of such problems can be used to calculate unsteady temperature fields and heat fluxes in two-layer sheet products, structures and buildings, in flat samples of products and apparatuses for which thermophysical parameters functionally depend on the temperature and the interface of the three-layer medium, and in addition, can be used for the calculation of unsteady temperature fields and heat fluxes in two-layer sheet products, structures and buildings, in flat samples of products and apparatuses for which thermophysical parameters functionally depend on the temperature and the interface of the three-layer medium, and in addition, can be used for the calculation of unsteady temperature fields and heat fluxes in the three-layer medium.The boundary condition of conjugation (of the fourth kind) corresponds to the heat exchange of the body surface with the environment according to the Fourier law of heat conduction or to the heat exchange of contacting solid bodies, when the temperature of the contacting surfaces is the same (ideal, perfect thermal contact), besides, the flowing of a solid body is determined by the flow of a liquid (or gas), the heat transfer from the liquid (gas) to the body surface in the immediate vicinity of the body surface occurs according to the law of heat conduction (molecular heat transfer), which has the following properties The problem in this formulation is considered for the first time. The practical value of the obtained results is determined by the solution of specific initial boundary value problems in a three-layer medium for the heat conduction equation.

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Hermite-Type Sampling Operator

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key-words: Generalized sampling operator, samplings of derivatives.

Abstract:
The generalized sampling operator is able to approximate bounded continuous functions and it is modeled on the sampling expansion for band-limited functions given by the Whittaker-Kotel'nikov-Shannon theorem. During the decades, some variations of this classical theorem have been proposed. One of them (dating back to Jagerman and Fogel and, in a more general form, to Linden and Abramson) takes into consideration also the derivative samples for the reconstruction of band-limited functions, with a consequent benefit of a larger sampling rate compared to the Whittaker-Kotel'nikov-Shannon theorem. Motivated by this new reconstruction, we modify the generalized sampling operator including the samplings of derivatives up to a generic order to approximate non necessarily band-limited functions. The definition of the new operator is

$$(G_{n,w}f)(x) = \sum_{k\in\mathbb{Z}} \left(\sum_{j=0}^{n} \frac{1}{j!} f^{(j)}\left(\frac{k}{w}\right)\left(x - \frac{k}{w}\right)^j \right) \chi(wx-k), \qquad x \in \mathbb{R},$$

(where χ is, as usually, a kernel) and we call it an *Hermite-type sampling operator* of order $n \in \mathbb{N}^+$. One of its main features is the faster order of approximation. Besides the convergence and its rate, we discuss well-posedness, regularity, simultaneous approximation and a Voronovskaya-type formula.
This talk is based on [1].

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Nonlinear Reduced Modeling for FitzHugh-Nagumo Equation: A Comparison of Kernel Functions

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key-words: Kernel principle component analysis, pattern formation, kernel functions.

Abstract:
Linear methods remain fundamental in data analysis due to their strong theoretical grounding and broad applicability. However, many real-world systems exhibit nonlinear behaviors that linear approaches cannot fully capture. This study presents a reduced-order modeling technique using Kernel Principal Component Analysis (KPCA), a nonlinear extension of traditional PCA, applied to solution data obtained from the discrete solution vectors of the FitzHugh-Nagumo (FHN) equation. Spatial and temporal discretizations are performed using the finite difference method and the semi-implicit Euler scheme, respectively. Multidimensional scaling (MDS) is employed for non-iterative reconstruction from reduced to full space. KPCA demonstrates the ability to preserve pattern behavior of the solutions of the FHN system, while enabling efficient low-dimensional approximations. Comparative analysis of various kernel functions reveals their substantial impact on model accuracy and efficiency, underscoring the importance of tailored kernel selection for capturing underlying data structures.

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Commutator of Sharp Maximal Operator on Total Morrey Spaces

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key-words: Sharp maximal operator, commutator, total Morrey spaces.

Abstract:
In this talk we consider the commutator of the sharp maximal operator $[b, M^\sharp]$ on the total Morrey spaces $L_{p,\lambda,\mu}(\mathbb{R}^n)$. We obtain necessary and sufficient conditions for the boundedness of the commutator of the sharp maximal operator $[b, M^\sharp]$ on the total Morrey spaces $L_{p,\lambda,\mu}(\mathbb{R}^n)$ when b belongs to the bounded space of mean oscillations $BMO(\mathbb{R}^n)$, thereby obtaining some new characterizations for some subclasses of $BMO(\mathbb{R}^n)$, see [1, 2, 3].

For $f \in L^1_{\rm loc}(\mathbb{R}^n)$, the sharp maximal operator $M^\sharp$ and the commutator of the sharp maximal operator $[b, M^\sharp]$ are defined by

$$M^\sharp f(x) = \sup_{r>0} |B(x,r)|^{-1} \int_{B(x,r)} |f(y) - f_{B(x,r)}| dy,\ [b, M^\sharp] f(x) = b(x) M^\sharp f(x) - M^\sharp(bf)(x),$$

where $B(x,r)$ is the ball of radius r centered at $x \in \mathbb{R}^n$ and $|B(x,r)|$ denotes the Lebesgue measure of $B(x,r)$.

Morrey spaces, introduced by C. B. Morrey 1938, play important roles in the regularity theory of PDE, including heat equations and Navier-Stokes equations. In [1] Guliyev introduce a variant of Morrey spaces called total Morrey spaces $L_{p,\lambda,\mu}(\mathbb{R}^n)$, $0 < p < \infty$, $\lambda \in \mathbb{R}$ and $\mu \in \mathbb{R}$, give basic properties of the spaces $L_{p,\lambda,\mu}(\mathbb{R}^n)$ and study some embeddings into the Morrey space $L_{p,\lambda,\mu}(\mathbb{R}^n)$, see also [2, 3]. Necessary and sufficient conditions for the boundedness of the maximal commutator operator M_b and the commutator of the maximal operator $[b, M]$ on $L_{p,\lambda,\mu}(\mathbb{R}^n)$ were also given. Some new characterizations for some subclasses of $BMO(\mathbb{R}^n)$ were obtained.

The aim of this work is to give necessary and sufficient conditions for the boundedness of the commutator of sharp maximal operator $[b, M^\sharp]$ on $L_{p,\lambda,\mu}(\mathbb{R}^n)$ when b belongs to $BMO(\mathbb{R}^n)$ spaces. New characterizations of some subclasses of $BMO(\mathbb{R}^n)$ spaces are obtained, see [2, 3].

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Commutator of Anisotropic Maximal Operator on Total Anisotropic Morrey Spaces

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key-words: Total anisotropic Morrey spaces, anisotropic maximal function, anisotropic fractional maximal function, commutator, anisotropic Lipschitz spaces.

Abstract:
In this talk we consider the commutators of the anisotropic maximal operator $[b, M^d]$ in total anisotropic Morrey spaces $L^d_{p,\lambda,\mu}(\mathbb{R}^n)$ when b belongs to anisotropic Lipschitz spaces $\dot{\Lambda}_{\beta,d}(\mathbb{R}^n)$.

Morrey spaces, introduced by C. B. Morrey 1938, play important roles in the regularity theory of PDE, including heat equations and Navier-Stokes equations. In [3] Guliyev introduce a variant of Morrey spaces called total Morrey spaces $L_{p,\lambda,\mu}(\mathbb{R}^n)$, $0 < p < \infty$, $\lambda \in \mathbb{R}$ and $\mu \in \mathbb{R}$, see also [4]. In [1] the authors was consider the total anisotropic Morrey spaces $L^d_{p,\lambda,\mu}(\mathbb{R}^n)$, give basic properties of the spaces $L^d_{p,\lambda,\mu}(\mathbb{R}^n)$ and study some embeddings into the Morrey space $L^d_{p,\lambda,\mu}(\mathbb{R}^n)$. Necessary and sufficient conditions for the boundedness of the anisotropic maximal commutator operator M^d_b and the commutator of the anisotropic maximal operator $[b, M^d]$ on $L^d_{p,\lambda,\mu}(\mathbb{R}^n)$ were also given. Some new characterizations for some subclasses of $BMO(\mathbb{R}^n)$ were obtained.

The aim of this work is to give necessary and sufficient conditions for the boundedness of the commutator of anisotropic maximal operator $[b, M^d]$ on $L^d_{p,\lambda,\mu}(\mathbb{R}^n)$ when b belongs to anisotropic Lipschitz spaces $\dot{\Lambda}_{\beta,d}(\mathbb{R}^n)$. New characterizations of some subclasses of anisotropic Lipschitz spaces $\dot{\Lambda}_{\beta,d}(\mathbb{R}^n)$ are obtained, see [2].

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Approximation Theorem via Statistical ψ-convergence on $H_\omega(K)$

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key-words: Positive linear operators, statistical convergence, the Bleimann, Butzer and Hahn operators.

Abstract:
In this study, a general Korovkin-type theorem is presented for sequences of positive linear operators from $H_\omega(K)$ to $C_B(K)$ where $K = [0, \infty)$. This theorem is proved using the concept of statistical ψ-convergence, which is a generalization of the concept of statistical convergence stated independently by Fast and Steinhauss ([2], [2]). Then, an example that satisfies the conditions of our main theorem will be presented. Finally, we calculate the corresponding rate of the statistical ψ-convergence of positive linear operators through the continuity modulus.

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Generalized $k-$Lauricella Functions

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key-words: Gamma and Beta k-function, Pochhammer k-symbol, k-Appell functions, k-Lauricella functions, generalized k-Lauricella functions.

Abstract:
The main purpose of this study is to introduce $k-$analogue of the Lauricella functions and the generalized Lauricella functions. Firstly, definition of k-Lauricella functions are defined with the help Pochhammer k-symbol. Later Generalized k-Lauricella functions are explained. Then theorems and implementations are given.

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A New Formula of Delta Laplace Transform for the Anti-causal System Defined on Isolated Time Scales

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key-words: Anti-causal system, Laplace transform, time scales.

Abstract:

This talk aims to present recent findings related to the unilateral and bilateral Laplace transforms for anti-causal systems defined on isolated time scales. The foundational work by Ortigueira et al. [4] introduced the Laplace transform for anti-causal systems, which was later generalized for arbitrary time scales by Ortigueira et al. [5] and San and Ortigueira [6]. We will compare our results with previously published findings by the authors listed in the bibliography, emphasizing the relevance and implications of our research in the context of existing literature.

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Construction of New Curves on nth Degree Polynomial-Type Curves Using the Flc Frame

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key-words: Pedal curves, Frenet like frame, polynomial curves.

Abstract:
In this study, first, the pedal curves were defined as the geometric locus of perpendicular projections onto the Frenet-like frame (Flc) vectors T, D_2 and D_1 of a given polynomial curve. Second, the Frenet elements of the new pedal curves were expressed in terms of Flc frame vectors T, D_2 and D_1, such that the interrelations among these frames could be established. Next the curvature and torsion functions of each pedal curve were examined, separately. Then, by considering the Frenet elements of these pedal curves as position vectors new Smarandache curves were defined. Furthermore, for each Smarandache curve, the expressions of curvature and torsion functions based on Frenet-like frame were obtained. Finally, new type of curves were introduced and contributed to the curve family, and most importantly an applet with animation was created and shared on the freeware Geogebra program for anyone to define new pedal curves according to Flc frame, simultaneously.

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Local Laplacian Equienergetic Graphs

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key-words: Graph energy, local Laplacian energy, local Laplacian equienergetic graphs.

Abstract:
Let G be a graph with vertex set $\{v_1, v_2, \ldots, v_n\}$ and L be the *Laplacian matrix* of G. The eigenvalues of L are called the *Laplacian eigenvalues* of G. Then, *Laplacian energy* of G is defined as

$$LE(G) = \sum_{i=1}^{n} |\lambda_i(L) - \bar{d}|,$$

where $\bar{d} = \frac{\sum_{i=1}^{n} d_i}{n}$ see [3, 4]. It is our main concern in this paper to study the problem of the variation of the energy when a vertex is deleted, which was first studied in Espinal and Rada [2], and more recently in [1, 5]. Let $G^{(v)}$ be the graph obtained from G by deleting vertex v. The quantity

$$\mathscr{LE}_G(v) = LE(G) - LE(G^{(v)})$$

is the variation of the Laplacian energy in G when the vertex v is deleted, and called as the *local Laplacian energy of G at vertex v*. Then, the *local Laplacian energy of G* is defined as:

$$e_L(G) = \sum_{v \in V(G)} \mathscr{LE}_G(v).$$

In this study, we define and characterize local-equienergetic graphs whose local Laplacian energies remain invariant under specific structural perturbations. In particular, we show that the deletion of an edge can preserve local Laplacian energy under certain conditions. Our results contribute to the broader understanding of energy-based invariants in spectral graph theory and offer new insights into the resilience of graph structures under local modifications.

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Approximation by Bivariate Modified Generalized Sampling Operators

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key-words: Generalized sampling series, rate of convergence, bivariate modulus of contunity.

Abstract:
In this study, we introduce a new family of bivariate generalized sampling operators by modifiying the classical operators through a function ρ that satisfies the suitable conditions. We investigate the pointwise and uniform convergence of our newly defined operator. We give the rate of convergence of the family of operators via the classical bivariate modulus on continuity. We also obtain an asymptotic formula in the sense of voronovskaja.
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Result for Multivalued Contractions on Ultrametric Spaces

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key-words: Fixed point, F-contraction, ultrametric space.

Abstract:
Recently, fixed point theory is studied on ultrametric space. The present talk deals with new fixed point theorems by F-contraction especially rational type and integral type mappings. The special cases of the results which reduce the existence are presented and some examples are given illustrate our results.

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Stability to Perturbations of *S*-fractions with Applications

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key-words: Stability to perturbations, continued fraction, convergence, relative errors.

Abstract:
This presentation is devoted to studying stability under perturbations for a class of functional continued fractions known as *S*-fractions (Stieltjes continued fractions) [4]. While these mathematical constructs are powerful tools in applied problems, their practical application requires a thorough understanding of their stability under perturbations. We establish formulas quantifying the relative errors of the approximants that result directly from perturbations applied to the elements of the continued fraction itself. Utilizing these formulas and leveraging techniques from the analytical theory of continued fractions, we derive sufficient conditions for the stability of these continued fractions under perturbations. A significant outcome of this research is the derivation of explicit bounds for the relative errors of the approximants, demonstrating how the perturbation error of a continued fraction's approximant depends on the magnitude of the perturbations and the values of the fraction's coefficients.

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On the Approximation of the Lauricella-Saran's Hypergeometric Functions F_K and Their Ratios by Branched Continued Fractions

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key-words: Hypergeometric function, branched continued fraction, approximation by rational functions, convergence, analytic continuation.

Abstract:
We consider the problem of approximating Lauricella-Saran's hypergeometric functions by a special family of functions – branched continued fractions. We establish domains of the analytical continuation of Lauricella-Saran's hypergeometric function F_K with real and complex parameters, using their branched continued fraction expansions whose elements are polynomials of three complex variables [1, 2]. To do this, we used the so-called extension theorem, which allows us to extend the domain of convergence of the branched continued fraction, which is already known for a small domain, to a larger domain, as well as the PC method (so called correspondence property) to prove that it is also the domain of analytical continuation. At the end, several numerical experiments are presented that illustrate the efficient approximation of special functions by branched continued fractions.

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Approximation by Kantorovich Sampling Operators in Weighted Variable Exponent Lebesgue Spaces

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key-words: Kantorovich sampling operator, direct estimate, Voronovskaya-type estimate, strong converse inequality, saturation, modulus of smoothness, weighted variable exponent Lebesgue space.

Abstract:
Bardaro, Butzer, Stens and Vinti [1] introduced the Kantorovich-type sampling operators

$$(S_w^\chi f)(x) := \sum_{k\in\mathbb{Z}} \frac{w}{\theta_k} \int_{t_k/w}^{t_{k+1}/w} f(u)\,du\,\chi(wx-t_k), \quad x\in\mathbb{R}, \quad w>0.$$

Here f is a locally Lebesgue integrable function, defined on $\mathbb{R}$, and $\chi:\mathbb{R}\to\mathbb{R}$ is the kernel of the sampling operator. The nodes $\{t_k\}_{k\in\mathbb{Z}}$ satisfy the assumptions $t_k < t_{k+1}$ and $\lim_{k\to\pm\infty} t_k = \pm\infty$, as, in addition, $\theta \le \theta_k := t_{k+1} - t_k \le \Theta$ for all $k\in\mathbb{Z}$ with some positive constants θ, Θ.

We will present a direct estimate of the norm of the error of S_w^χ by a modulus of smoothness in certain variable exponent Lebesgue spaces with the weight

$$\rho_{\alpha,\beta}(x) := \begin{cases} |x|^{-\alpha}, & x<-1, \\ 1, & -1\le x\le 1, \\ x^{-\beta}, & x>1, \end{cases}$$

where $\alpha,\beta\ge 0$. The variable exponent is such that the Hardy-Littlewood maximal operator is bounded.

Further, we will complement the direct error estimate with a converse one, which enables us to establish the saturation property of this approximation process and describe its saturation class and its trivial class.

The results to be presented have recently appeared in [2].

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On λ_f-Statistical Convergence of Sequences of Sets

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key-words: Modulus function, Wijsman statistical convergence, statistical convergence.

Abstract:
In this work, using an unbounded modulus function f we introduce and examine the concepts of Wijsman λ_f-statistical convergence and Wijsman strongly λ_f-convergence, and give some relations between Wijsman λ_f-statistical convergence and Wijsman strongly λ_f-convergence.

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On the Least Squares and Weighted Least Squares Estimation Based on Ranked Set Sampling

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key-words: Rank set sampling, estimation, perfect ranking, Monte Carlo simulation order statistics.

Abstract:
Rank set sampling stands out as an effective and practical method for parameter estimation, especially in cases where complete and accurate measurement of observations is difficult or costly. In this study, the least squares and weighted least squares methods is consered based on order statistics and used in estimating the parameters of the Poisson new XLindley distribution. An extensive Monte Carlo simulation study is performed to compare the proposed least squares and weighted least squares methods with classical estimation methods. The results of the simulation studies show that the proposed modification has lower mean squared errors compared to the estimation methods used in classical rank set sampling, especially in the case of small samples. Applications carried out on real data sets also support the theoretical findings. As a result, estimators based on ranked set sampling offer a strong alternative in terms of obtaining more robust and reliable results in statistical modeling and inference studies.
Acknowledgements: The first author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The author thanks to TUBITAK for their supports.

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3rd International Conference: Constructive Mathematical Analysis

Trees Families with Algebraic Connectivity Greather than or Equal to $2(1 - cos(\pi/9))$

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key-words: Trees, Laplacian matrix, algebraic connectivity.

Abstract:
For a simple and undirected graph G with n vertices and adjacency matrix A. The Laplacian spectrum of G is set of eigenvalues $\lambda_n(G) \geq \lambda_{n-1}(G) \geq \ldots \geq \lambda_2(G) \geq \lambda_1(G) = 0$ of the Laplacian matrix L of G, given by $L = D - A$, where D is the diagonal matrix of vertex degrees. The second smallest eigenvalue of the Laplacian matrix is called algebraic connectivity and is often denoted by $\alpha(G)$ in the literature.

Let T be a tree (a connected acyclic graph) on n vertices and $\alpha(T)$ be algebraic conncectivity of T. Classifying trees according to their algebraic connectivity values has been an interesting area of study. Yuan et al. classified trees such that $\alpha(T) \geq 2 - \sqrt{3}$ [2], Wang and Tan classified trees such that $\alpha(T) \geq \frac{5-\sqrt{21}}{2}$ [3]. Later, Belay et al. showed tree classes such that $\alpha(T) \geq 2(1 - cos(\pi/7)))$ [1]. The lower bounds $2 - \sqrt{3}$, $\frac{5-\sqrt{21}}{2}$ and $2(1 - cos(\pi/7))$ given for algebraic connectivity in [1, 2, 3] were characterized actually by Kirkland in [4]. In this study, we give new limit points in a similar way and find new tree classes with algebraic connectivity greater than or equal to $2(1 - cos(\pi/9))$.

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Fuzzy Soft Element and Its Application to Decision-making

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key-words: Fuzzy soft set, fuzzy soft element, decision-making.

Abstract:
This paper proposes the novel concept of fuzzy soft elements, thereby extending the traditional notion of fuzzy soft points through the more flexible assignment of alternatives to attributes. Consequently, this concept offers a new vantage point for understanding fuzzy soft set operations. Through this, after some of the properties of fuzzy soft set operations are given, a decision-making example is presented that demonstrates the practical application.

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New Fixed Point Theorem for Rational F-Contractions

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key-words: Fixed point, rational type, metric space.

Abstract:
In [1], Wardowski introduced the concept of F-contraction, proving that every F-contraction mapping on a complete metric space has a unique fixed point, with consecutive approximations converging to this fixed point. For the function $F = \ln(x)$ on the interval $x \in (0, 1)$, a Banach contraction is an example of an F-contraction, which extends the main theorem of [1] to the Banach Fixed Point Theorem. There are many papers have been studied on this subject set [1, 2]. The most evident results are concentrated on applications of F-contraction to differential, integral equation, homotopy,... In this talk, we extend Wardowski's technique to present fixed point results for rational type F-contraction mappings in spaces equipped with two metrics.
Acknowledgements: The first author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The author thanks to TUBITAK for their supports.

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A Theoretical Approach to Superfluous Parameters in Constrained Multivariate Linear Models

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key-words: BLUP, multivariate linear models, superfluous parameters.

Abstract:
In this study, we develop a comprehensive methodology for analyzing constrained multivariate linear models (CMLMs) with superfluous parameters. The proposed model is constructed by augmenting new regressors to a base constrained multivariate linear model. Our analysis considers both the original constrained model and its extended version with superfluous parameters. With these models, the constraint added to the models is written in a common matrix form in terms of unknown parameters and converted to their unconstrained equivalents, thus providing an easier to process analysis. We derive explicit analytical formulas for computing the best linear unbiased predictors (BLUPs) and best linear unbiased estimators (BLUEs) of all unknown parameter matrices by solving systems of linear equations. The properties and analytical expressions of these predictors and estimators are examined within the framework of both models. In particular, several novel and valuable properties of BLUPs are established. Additionally, we provide analytical solutions for special cases involving the general vector of all unknown parameter vectors. For methodologies and related models, see [1] and [2, 3], respectively.

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d(d^{-1})-Continuous Multi-valued Mappings and Some Fixed Point Results

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key-words: Quasi-metric, fixed point, d-continuity, d^{-1}-continuity, multi-valued map.

Abstract:
Banach contraction theorem [1] has been examined and improved by many researcher due to its applicability to many fields. In [4], Nieto et al. obtained a new extension of Banach contraction mapping theorem to partially ordered sets. The concept of quasi-metric space was introduced by Wilson [6]. In [5] , Schellekens showed that quasi-metric spaces are very suitable tools for analyzing the complexity analysis of algorithms. Recently, in [2] new fixed point results in partially ordered bicomplete quasi-metric spaces have been obtained. In this work, we introduce the concept of d-continuity and d^{-1}-continuity for multivalued mappings defined on a partially ordered quasi-metric spaces and we prove some fixed point theorems for multivalued maps. Moreover, we give an application of our results to homotopy theory.

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Non-parametric Estimation for Doubly Geometric Process

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key-words: Geometric process, doubly geometric process, non-parametric estimation, least square method, inverse Gaussian distribution.

Abstract:
The geometric process (GP) is a widely used stochastic model in the reliability theory. Lam [1, 2] first introduced the model as a generalization of the renewal process (RP) and used the process as a model in the reliability context. Lam [5] introduced the GP and its applications. Although the GP is an important and widely used model in many areas, it has some limitations. Wu [4] proposed the doubly geometric process (DGP) as an alternative model that can overcome the limitations of the GP. The parameter estimation problem is very important for both GP and DGP. Lam [3] introduces a least square (LSE) method for nonparametric (NP) estimation problem in GP. In this study two NP methods proposed by Jasim and Al-Qazaz [1] for DGP which are (LSE) and log-LSE are presented. Additionally, modified moment (MM) estimators are obtained. The efficiencies of the maximum likelihood (ML) estimators are compared with (MM) estimators by an extensive simulation study.

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Parameter Estimation for Doubly Geometric Process with the Inverse Gaussian Distribution

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key-words: Geometric process, doubly geometric process, parametric estimation, maximum likelihood method, inverse Gaussian distribution.

Abstract:
The geometric process (GP) is a widely used stochastic model in the reliability theory. Lam [1, 2] first introduced the model as a generalization of the renewal process (RP)and used the process as a model in the reliability context. Lam [3] introduced the GP and its applications. Although the GP is an important and widely used model in many areas, it has some limitations. The two limitations can be given as follows: The GP cannot be used for non-monotone interarrival times whose distributions have varying shape parameters. The second limitation is that only logarithmic growth or explosive growth is allowed. Wu [4] proposed the doubly geometric process (DGP) as an alternative model that can overcome the limitations of the GP. The parameter estimation problem is very important for both GP and DGP. In this study, the parameter estimation problem for a DGP is consider under the assumption that the first inter-arrival time has inverse Gaussian (IG) distribution. The estimators of the parameters are obtained by maximum likelihood method. Additionally, the asymptotic unbiasedness and consistency of the estimators are investigated. The asymptotic distributions of the maximum likelihood (ML) estimators are obtained and some statistical properties such as unbiasedness and consistency of the estimators are investigated. It is shown in the Monte Carlo simulation study that the ML estimators are highly efficient estimators.

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Quasi-metric Spaces and Related Fixed Point Results

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key-words: Quasi-metric space, fixed point.

Abstract:
We present a series of further results, in different settings, examining the fixed point results for such spaces.

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On the Two-layer Adjoint Problem for the Heat Conduction Equation in a Rectangle

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key-words: Two-layer medium, temperature field, unsteady heat conduction, method of integral transformations, conjugation conditions.

Abstract:
The analytical theory of unsteady heat conduction in a rectangular plate finds wide application in solving important technical and technological problems expressing Fourier's and Fick's laws at the corresponding dependence of the internal energy of the medium on temperature. In the article the initial boundary value problem for the equation of heat conduction in a rectangular plate with a given initial temperature distribution, boundary conditions and conjugation conditions on the break line of the rectangular plate are considered. Thermodynamic processes are accompanied by heat transfer between elements of structures in the working space of thermal plants and the surrounding space, in which heat is a quantitative measure of energy and between bodies there is heat exchange and regularities of the process of heat transfer are considered by the theory of heat exchange and heat transfer. The analytical solution of the problem of unsteady heat conduction in a two-layer medium is constructed by the method of integral transformations: Laplace on the time variable and finite Fourier transform on the coordinate variables. The study of initial boundary value problems for the heat conduction equation and the creation of analytical methods for solving such problems represent an important and rapidly developing area of mathematical physics The solution of such problems can be used to calculate unsteady temperature fields and heat fluxes in two-layer sheet products, structures and buildings, in flat samples of products and apparatuses for which thermophysical parameters functionally depend on the temperature and the interface of the two-layer medium, and in addition can be used to calculate unsteady temperature fields and heat fluxes in two-layer sheet products, structures and buildings. The boundary condition of conjugation (of the fourth kind) corresponds to the heat exchange of the body surface with the surrounding medium according to the Fourier law of heat conduction or to the heat exchange of adjoining solid bodies, when the temperature of the adjoining surfaces is the same (ideal, perfect thermal contact), besides, the flowing of a solid body is determined by the flow of liquid (or gas), the heat transfer from liquid (gas) to the body surface in close proximity to the body surface occurs according to the law of heat conduction (molecular heat transfer), which has the following properties

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On Controlled Interpolative Metric Spaces

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key-words: Graphical interpolative metric space, controlled metric space, Kannan contraction.

Abstract:
Recently, Karapinar [1] introduced a new generalized metric space (so called interpolative matric space). After than, Panda [2] generalized this concept which is called extended interpolative metric spaces. Finally, Girgin et.al. [3] defined the controlled interpolative metric spaces. This study introduces new type fixed point theorems using Kannan contraction in the novel framework of controlled interpolative metric spaces.

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Commutator of Sharp Maximal Operator on Lorentz Spaces

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key-words: Maximal operator, commutator, Lorentz space.

Abstract:
In this talk we consider the commutator of the sharp maximal operator $[b, M^\sharp]$ on the Lorentz spaces $L^{p,q}(\mathbb{R}^n)$. We obtain necessary and sufficient conditions for the boundedness of the commutator of the sharp maximal operator $[b, M^\sharp]$ on the Lorentz spaces $L^{p,q}(\mathbb{R}^n)$ when b belongs to the bounded space of mean oscillations $BMO(\mathbb{R}^n)$, thereby obtaining some new characterizations for some subclasses of $BMO(\mathbb{R}^n)$, see [1, 2].

For $f \in L^1_{\rm loc}(\mathbb{R}^n)$, the maximal operator M and the commutator of the anisotropic sharp maximal operator $[b, M^\sharp]$ are defined by

$$M^\sharp f(x) = \sup_{r>0} |B(x,r)|^{-1} \int_{B(x,r)} |f(y) - f_{B(x,r)}| dy, \ [b, M^\sharp] f(x) = b(x) M^\sharp f(x) - M^\sharp(bf)(x),$$

where $B(x,r)$ is the ball of radius r centered at $x \in \mathbb{R}^n$ and $|B(x,r)|$ denotes the Lebesgue measure of $B(x,r)$. Suppose that f is a measurable function on $\mathbb{R}^n$, then we define

$$f^*(t) = \inf\{s > 0 : d_f(s) \le t\}, \quad d_f(s) := |\{x \in \mathbb{R}^n : |f(x)| > s\}|, \quad \forall s > 0.$$

The Lorentz space $L^{p,q} \equiv L^{p,q}(\mathbb{R}^n)$, $0 < p, q \le \infty$ is the collection of all measurable functions f on $\mathbb{R}^n$ such the quantity

$$\|f\|_{L^{p,q}} := \|t^{\frac{1}{p}-\frac{1}{q}} f^*(t)\|_{L^q(0,\infty)}$$

is finite. Clearly $L^{p,p} \equiv L^p$ and $L^{p,\beta} \equiv WL^p$. The functional $\|\cdot\|_{L^{p,q}}$ is a norm if and only if either $1 \le q \le p$ or $p = q = \infty$.

Theorem 1 *Let $p, q \in (1, \infty)$. The following assertions are equivalent:*

(i) $b \in BMO(\mathbb{R}^n)$ *and* $b^- \in L^\infty(\mathbb{R}^n)$.

(ii) *The operator* $[b, M^\sharp]$ *is bounded on* $L^{p,q}(\mathbb{R}^n)$.

(iii) *There exist a constant* $C > 0$ *such that* $\sup_B \dfrac{\left\| \left(b(\cdot) - 2M^\sharp(b)(\cdot)\right)\chi_B \right\|_{L^{p,q}(\mathbb{R}^n)}}{\|\chi_B\|_{L^{p,q}(\mathbb{R}^n)}} \le C.$

(iv) *There exist a constant* $C > 0$ *such that* $\sup_B |B|^{-1} \left\| \left(b(\cdot) - 2M^\sharp(b)(\cdot)\right)\chi_B \right\|_{L^1(\mathbb{R}^n)} \le C.$

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Comparison of Various Types of Inverse Soft Covering Upper Approximations

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key-words: Inverse soft set, rough set, inverse soft rough set, decision making.

Abstract:
This study presents a comparison of various inverse soft covering upper approximations, focusing specifically on the differences between them. A key finding of our work is the introduction of a new type of inverse soft covering upper approximation which is a new approach in developing inverse soft covering based rough sets. We demonstrate that this new approximation is smaller than the three existing approximations described in the literature. Based on this result, we aim to manage the decision-making processes, especially for the uncertainty problems, in a more ideal way.

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Generalizations of Certain Classes of Single-Valued Mappings to Multivalued Cases

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key-words: Fixeds point, multivalued mappings, existence.

Abstract:
In this study, multivalued generalizations of certain classes of single-valued transformations defined on metric spaces are obtained. Building upon recently introduced concepts such as mappings contracting perimeters of triangles, new structural properties and fixed-point results for the multivalued counterparts are explored. Further examples and applications are also provided to illustrate the theoretical framework.

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On Groups and Vector Spaces with Easy Direct Limits

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key-words: Algebra, direct limit, retract, group, vector space.

Abstract:
The direct limit construction belongs to basic tools used in Universal algebra, see e.g. [2]. Despite its frequent application, a systematic account of properties of this construction is rare in the literature, for a recent one see [1].

We will focus on direct limit families in which only one algebra occurs. Groups and vector spaces will be considered as algebras.

Let $\mathcal{A}$ be an algebra. We denote by $\underrightarrow{\mathbf{L}}\mathcal{A}$ the class of all isomorphic copies of direct limits which can be obtained from $\mathcal{A}$ and we denote by $\mathbf{R}\mathcal{A}$ the set of all retracts of $\mathcal{A}$. Then $\mathbf{R}\mathcal{A} \subseteq \underrightarrow{\mathbf{L}}\mathcal{A}$. We will say that $\mathcal{A}$ *has easy direct limits* if every algebra from $\underrightarrow{\mathbf{L}}\mathcal{A}$ is isomorphic to a retract of $\mathcal{A}$.

If $\mathcal{A}$ is finite, then $\mathcal{A}$ is an algebra with easy direct limits, cf.[3]. Infinite mono-unary algebras with easy direct limits are studied in [4].

We will present a mono-unary approach which helps to recognize algebras with easy direct limits. It works for an algebra whose some term operation is also an endomorphism. E.g., for multiplicative group of non-zero rational numbers we can use the term operation x^2 to see that this group is not with easy direct limits. Further, we show that

- every finite dimensional vector space has easy direct limits.
- the additive group of rational numbers has easy direct limit and additive groups of integers and real numbers do not.

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Solution of Split Equality Fixed Point Problem for Finite Family of Demigeneralized Mappings

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key-words: Split equality fixed point problem, demigeneralized mappings, weak convergence.

Abstract:
In this paper, we constructed and study an iterative algorithm for approximating solutions of split equality fixed point problem (SEFPP) for finite family of demigeneralized mappings in uniformly convex and uniformly smooth real Banach spaces. Weak convergence of the sequence generated by the algorithm was established. Strong convergence theorem was also proved under the assumption that the operators are demi-compact. Moreover, we give application of our result to split equality problem (SEP)

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Preserving Non-Negativity in Rational Approximation of Fractional Diffusion Problems

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key-words: Fractional diffusion, rational approximation, non-negativity preserving.

Abstract:
Maximum principles are among the basic qualitative properties of the elliptic or parabolic operators. In numerically solving PDEs, it is a natural requirement that these properties be preserved at the discrete level.

The presented analysis is dedicated to problems in a multidimensional domain Ω with general geometry. Let $\mathcal{A}$ be a self-adjoint elliptic operator of second order. Our study is focussed on the case of spectral fractional diffusion where $\mathcal{A}^\alpha = \mathcal{W}^T \mathcal{D}^\alpha \mathcal{W}$. Here $\mathcal{W}$ is the row-matrix of eigenfunctions and $\mathcal{D}$ is the diagonal matrix of eigenvalues of $\mathcal{A}$. Let the symmetric and positive definite matrix A be obtained after a finite difference or finite element discretization of $\mathcal{A}$.

We first discuss the BURA (*Best Uniform Rational Approximation*) method for $\alpha \in (0,1)$ (sub-diffusion), which is introduced in [2]. At the discrete level, the method is defined in the form $A^{-\alpha} \approx \mu_1^{-\alpha} r_{\alpha,k}(\mu_1 A^{-1})$. Here $r_{\alpha,k}(z)$ is the best uniform rational approximation of degree (k,k) to z^α, $z \in [0,1]$ and $\mu_1 > 0$ is the minimal eigenvalue of A. Following [1] we further develop sufficient conditions for positive approximation of the operator $A^{-\alpha}$. Next, we investigate the case of $\alpha > 1$ (super diffusion). Here, the conditions for non-negativity preservation require a more careful analysis on both BURA and BURA-based methods (see, e.g. [3]).

The last part of the talk is devoted to the analysis of parabolic problems. In time stepping, θ-methods are utilised with $\theta \in \{0, 0.5, 1\}$. For implicit schemes, we need BURA or a BURA-based approximation of A^α. It may be surprising to some readers that numerically multiplying a vector by A^α is a more complicated task than solving the corresponding fractional diffusion linear system. A significant part of the presented results concerns the dependence of the non-negativity conditions on the fractional power of the diffusion operator.

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Anti-Periodic Boundary Value Problems for Singular Fractional p-Laplacian Equations

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key-words: Anti-periodic boundary value problems, Caputo fractional differential equations, fractional p-Laplacian, fixed point theorem, global existence.

Abstract:
We study the following anti-periodic boundary value problems for Caputo fractional differential equations with a singular nonlinearity involving p-Laplacian operator ([1], [2], [3], [4], [5], [6] and [7]).

$$\begin{cases} D_*^{\beta}\phi_p(D_*^{\alpha}x(t)) = t^{-\gamma}f(t,x(t)), \\ x(0) = -x(1),\ D_*^{\alpha}x(0) = -D_*^{\alpha}x(1), \\ x \in Dom(D_*^{\alpha}),\ \phi_p(D_*^{\alpha}x) \in Dom(D_*^{\beta}),\ D_*^{\alpha}x \in C[0,1], \end{cases} \tag{1}$$

where $f \in C([0,1]\times\mathbb{R})$, $0 \le \gamma < \alpha$, $\beta \le 1$, $\phi_p(t) = |t|^{p-2}t$, $p > 1$. Our main goal is to prove the existence of a solution to the problem (0.1). We first, show that this problem is equivalent to a class of Volterra integral equations.

Theorem 1 *Let $f \in C([0,1]\times\mathbb{R})$ and $0 \le \gamma < \alpha$, $\beta \le 1$. A function $x \in C[0,1]$ is a solution of the anti-periodic boundary value problem (0,1) if and only if $x \in C[0,1]$ satisfies the Volterra integral equation*

$$x(t) = I^{\alpha}\phi_q\big[I^{\beta}(t^{-\gamma}f(t,x(t))) + L_f x(t)\big] + M_f x(t). \tag{2}$$

where $L_f x(t) = -\frac{1}{2}I^{\beta}(t^{-\gamma}f(t,x(t)))\big|_{t=1}$ and $M_f x(t) = -\frac{1}{2}I^{\alpha}\phi_q\big[I^{\beta}(t^{-\gamma}f(t,x(t))) + L_f x(t)\big]\big|_{t=1}$.

The main result regarding the existence of the solution is given in the following theorem.

Theorem 2 *Let $f \in C([0,1]\times\mathbb{R})$ and there exist nonnegative functions $a, b \in C[0,1]$ such that $|f(t,u)| \le a(t) + b(t)|u|^{p-1}$, for all $t \in [0,1]$ and $u \in \mathbb{R}$. If $\frac{3^p}{2^p\Gamma(\alpha+1)^{p-1}}\frac{\Gamma(1-\gamma)}{\Gamma(1-\gamma+\beta)}\|b\| < 1$ then the Volterra integral equation (0.2) has a solution in $C[0,1]$.*

Finally, we give an example as an application of Theorem 0.2.

Example 0.1 *If $0 < b < \frac{1}{14}$ then the following problem has a solution in $C[0,1]$.*

$$D_*^{\frac{3}{4}}\phi_3(D_*^{\frac{1}{2}}x(t)) = t^{-1/3}(1 + bx^2(t)),$$
$$x(0) = -x(1),\ D_*^{\alpha}x(0) = -D_*^{\alpha}x(1).$$

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Squared Basis Operators Related to Modified Bessel Functions

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key-words: Squared basis, Bessel function, Voronovskaya-type results.

Abstract:

In our talk, we propose considering approximation properties of positive linear operators related to the modified Bessel function of the first kind. We focus on these operators because the operators related to the modified Bessel function of order 0 come in the King-type operators associated with the squared Szász-Mirakyan basis.
Similar to, for example, the case of Bernstein operators, it can be proved that the error of approximation by our operators is smaller than the classical Szász-Mirakyan ones.

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Non-Instantaneous Impulsive Differential Equations with a Caputo Fractional Derivative with respect to Other Functions and Strict Stability

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key-words: Strict stability, noninstantaneous impulses, Caputo derivatives with respect to another function, fractional differential equations, Lyapunov functions.

Abstract:
A nonlinear system of fractional differential equations with noninstantaneous impulses is considered. Caputo fractional derivatives with respect to others function (CFDF) are applied (see,[1]). The main characteristics of these fractional derivatives are two: first, the lower limit of CFDF changes at any starting time of action of the noninstantaneous impulses, the applied function in CFDF is changeable at each interval without impulses. .The strict stability for the studied system is defined (see, for example, [2, 3, 4]) As an auxiliary system we consider a system of two linear scalar noninstantaneous impulsive fractional equations with CFDF and define a strict stability in couple. The introduced definitions are illustrated on several examples. We use appropriate Lyapunov functions to obtain sufficient conditions for strict stability of the studied system. These sufficient conditions depends significantly on the type of the impulsive functions.
Acknowledgments: This research was partially funded by Bulgarian National Science Fund under Project KP-06-N62/1.

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Analysis of the mKdV Equation in Magnetohydrodynamics for Exploring Solitons and Nonlinear Dynamics

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key-words: The mKdV equation, the φ^6-model expansion technique, soliton solution.

Abstract:
In this study, we construct the traveling wave solutions for the nonlinear mKdV equation using the φ^6-model expansion approach. The KdV equation is applied in shallow water and stratified flows, classical and magnetohydrodynamics, plasma physics and other scientific fields. The explicit retrieval of a wide range of solution types, including bright, singular, dark, and singular soliton solutions, is made possible by the φ^6-model expansion technique. Another kind of solution that are explicitly retrieved are kink-type solitons, which are referred to as topological solitons in the context of water waves. The nonlinear dynamical features of the equation may be improved by the outcomes of this research. The method proposes a useful and effective approach for solving a large class of non-linear partial differential equations. Interesting graphs are used to illustrate and highlight the dynamical characteristics of the solutions obtained.

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Fixed Points in $\Delta-$symmetric Quasi Modular Metric Spaces of Non-Archimedean Type

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key-words: Non-Archimedean quasi modular metric spaces, $\Delta-$symmetry, fixed point theorems.

Abstract:
This study aims to present a novel and transformative concept of a non-Archimedean $\Delta-$symmetric quasi modular metric space, which represents a significant generalization that effectively integrates the characteristics of quasi metrics, modular metrics, and $\Delta-$symmetric frameworks. By employing this innovative structure, we derive Matkowski-type fixed point theorems under relaxed contractive conditions, thereby extending classical findings into a more intricate and asymmetric realm. Furthermore, we undertake a comprehensive analysis of the topological and convergence properties elicited by quasi modular metrics, revealing their complex interrelationships with traditional metric topologies. Our approach not only pushes the boundaries of existing research in fixed point theory but also significantly broadens its scope by removing continuity assumptions and incorporating variable iterations within the transformative context of the non-Archimedean and $\Delta-$symmetric paradigm.

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3rd International Conference: Constructive Mathematical Analysis

A Comparative Stability Analysis of Fixed Point Iterations

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key-words: Stability, fixed point, functional analysis.

Abstract:
In this work, we conduct a comparative stability analysis of several well-known fast iterative processes for contractive mappings in Banach spaces. Specifically, we examine the M-iteration and Kadıoğlu and Yildirim iteration processes providing a theoretical assessment of their stability conditions and convergence rates. The theoretical findings are supported by numerical experiments to illustrate their practical performance. Our results highlight how these iterative methods respond to variations in contraction parameters and initial conditions, offering valuable insights for selecting appropriate approaches in applied mathematics and computational sciences.

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Benchmarking Iterative Processes on Contractive Mappings with Numerical Applications

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key-words: Rate of convergence, fixed point, M-iteration.

Abstract:
In this study, we compare the convergence speed of several iterative methods for contractive mappings, focusing on the M-iteration and Kadıoğlu–Yıldırım iteration. Using Berinde's definition, we theoretically and numerically show that these methods converge faster than classical ones such as Picard, Mann, and Ishikawa. The findings help identify more efficient schemes for solving fixed point problems.

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Remark on Neumann Problem in the Ring Domain

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key-words: Neumann problem, ring domain, polyanalytic functions.

Abstract:
Recently, several complex model partial differential equations, which hold significant importance in various applied science fields, have been thoroughly examined, particularly in relation to the Neumann problem (see [1]-[3]). Additionally, numerous mathematicians have explored the solvability and solutions of complex partial differential equations under various boundary conditions (see [3]). This paper addresses the Neumann boundary value problem for harmonic functions in a ring-shaped domain within the complex plane. We investigate the existence and uniqueness of solutions to this problem by employing methods from complex analysis and potential theory. The problem is formulated with prescribed normal derivatives on the inner and outer boundaries of the annular domain. Using techniques such as Cauchy Pompeiu Representation Formula and Green's function representation, we derive explicit solution formulas and establish the necessary conditions for the solvability of the problem.
Acknowledgements: The author has been supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under the 2224-B Grant Program for Participation in Scientific Meetings within the Country. The author thanks TUBITAK for their support.

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Continued Fractions in Solving Congruences and Diophantine Equations

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key-words: Fractional differential equations, Riemann-Liouville operators, α-trigonometric functions, mittag-leffler functions, oscillatory solutions, memory effects.

Abstract:
Continued fractions provide an effective framework for representing real numbers and functions. Historically, algorithms such as Euclid's algorithm were employed to address number-theoretic problems, including the solution of Diophantine equations and congruences. This paper explores the historical development and mathematical foundations of continued fractions, emphasizing their role in solving linear Diophantine equations and congruences. We demonstrate how simple continued fractions can be utilized to find rational approximations and solutions to such equations. In particular, we show that a number aa has a multiplicative inverse modulo mm if and only if specific conditions related to continued fractions are satisfied.

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Approximation by a Generalization of the Szász-Baskakov Operators

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key-words: Szász-Baskakov operators, modulus of continuity, Voronovskaya-type theorem.

Abstract:
In this work we obtain a generalization of Szász-Baskakov operators with the help of special polynomials. The moments of the new operator are calculated. For our new operator, an approximation ratio is found by using the first and second modulus of continuity. Also, an asymptotic approximation is obtained by utilizing Voronovskaya-type theorem. Finally, error tables are given with the help of the determined functions.

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Surface Pencil along a Common Spatial Geodezic According to Positional Adapted Frame in Euclidean 3-space

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key-words: Surface pencil, positional adapted frame, Euclidean space

Abstract:
In this study, surface families that pass through a given space curve and consider this curve as a geodesic investigated. Using the positional adapted frame vectors of the curve, the surface family will be constructed as a linear combination of the components of the local coordinate frame, and the necessary and sufficient conditions examined. The surface families obtained theoretically will be supported with examples and visualized using drawing software. Finally, the geometric properties of the resulting new equations analyzed.

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Convergence of Steklov Type Neural Network Operators

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key-words: Neural networks, Steklov integrals, uniform convergence.

Abstract:
In this talk, we deal with construction of newly family of Neural Network operators, that is, Steklov Neural Network operators. By using Steklov type integral, we introduce a new version of Neural Network operators. We obtain some convergence theorems for the family, such as, pointwise and uniform convergence, rate of convergence via modulus of continuity.
Acknowledgements: The first author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The author thanks to TUBITAK for their supports.

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Quantitative Convergence and Multivariate Extensions of Activated Convolution Operators via Positive Linear Methods

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key-words: Trainable half-hyperbolic tangent function, positive linear operator, multidimensional convolution operator, simultaneous approximation.

Abstract:
In this work, we study trainable half-hyperbolic tangent function and convolution-based operators enhanced with activation mechanisms. The work highlights the importance of the "positive linear operators method" in understanding operator behavior and learning dynamics. Using the modulus of continuity, we derive quantitative convergence results towards the identity operator. We also extend the framework to multivariate settings by analyzing both simultaneous and iterative generalizations.

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Stability and Solution of Triple i-f-Hom-Der on Triple Banach Algebras

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key-words: System of additive-quadratic functional equations, triple i-f-hom-der, Hyers-Ulam stability.

Abstract:
In this work, we introduce a new concept of additive and quadratic equations, a type of system of additive-quadratic functional equations on triple Banach algebras. In the following, we define triple i-f-homomorphism-derivation (briefly, triple i-f-hom-der), where $i \in \{1, 2\}$ associated with the additive-quadratic equation where f is a triple linear homomorphism for $i = 1$ and a triple quadratic homomorphism for $i = 2$ on triple Banach algebras. Finally, we employ a fixed point approach to explore the stabilities of the Hyers-Ulam property for system of additive-quadratic functional equations and triple i-f-hom-der, utilizing two control functions from Găvruta and Rassias on triple Banach algebras.

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An Inverse Source Problem for Time-fractional Pseudoparabolic Equations with Memory Term

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key-words: Inverse problem, Caputo-type fractional derivative, pseudoparabolic equation.

Abstract:
Let Ω be a bounded domain in R^d, $d \geq 1$, with a smooth boundary and $Q_T := \Omega \times [0,T]$ is a bounded cylinder $0 < T < \infty$. In this work, we consider the following inverse problems for the pseudoparabolic equation with time-fractional Caputo derivative

$$\mathcal{D}_t^{\alpha}\left(u - \kappa\Delta u\right) - \beta\Delta u - \gamma \int_0^t K(t-\tau)\Delta u(x,\tau)d\tau = f(x)g(x,t) + h(x,t), (x,t) \in Q_T, \tag{1}$$

which is supplemented with the initial condition

$$u(x,0) = u_0(x), \quad x \in \Omega \tag{2}$$

and the boundary condition

$$u(x,t) = 0, \quad t \in [0,T], \quad x \in \partial\Omega, \tag{3}$$

and one of the following overdetermination condition

$$u(x,T) = a(x), \quad x \in \Omega \tag{4}$$

or

$$\int_0^T u(x,t)w(t)dt = a(x), \quad x \in \Omega. \tag{5}$$

Inverse problem I: Finding a pair of functions $\{u(x,t), f(x)\}$ that satisfy (1)-(3) and the final overdetermination condition (4).

Inverse problem II: Finding a pair of fucntions $\{u(x,t), f(x)\}$ that satisfy (1)-(3) and the integral overdetermination condition (5).
Here, the functions $K(t), g(x,t), u_0(x), a(x)$ and $h(x,t)$ and the constants $\kappa > 0, \beta > 0$ and $|\gamma| < \infty$ are given. In this talk, we study the questions of existence and uniqueness of a strong generalized solution of (1)-(4) or (1)-(3),(5).
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Analysis of Approximation Processes for Fuzzy Numbers via Truncated Max-Product Operators

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key-words: Max-product operators, shape characteristics, fuzzy numbers.

Abstract:
In this paper, we introduce a class of truncated max-product type operators defined over arbitrary compact intervals and investigate their approximation properties. It is demonstrated that these operators maintain the same order of uniform approximation, along with preserving monotonicity and shape characteristics, similarly to the classical constructions on unbounded domains. Additionally, comparisons and graphical illustrations are presented to emphasize the effectiveness of the proposed operators in the approximation of fuzzy functions.

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Some Basic Results of Eccentricity Sombor Index

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key-words: Eccentricity, lower bounds, upper bounds.

Abstract:
This paper is related to investigating the upper and lower bounds of eccentricity Sombor index, stated as $SO_{ec}(\mathcal{G})$. In mathematical terms, this index is defined as $SO_{ec}(\mathcal{G}) = \sum_{uv \in E(\mathcal{G})} \sqrt{ec^2(u) + ec^2(v)}$ where $E(\mathcal{G})$ is the set of edges in the graph $\mathcal{G}$, and $ec(.)$ is the eccentricity of a vertex of the edge in $E(\mathcal{G})$. The eccentricity of a vertex is defined as the maximum distance between the vertex and any other vertex in the graph. By analyzing graph structural aspects, the work provides new perspectives on the features and extremal behaviors of this graph invariant. The calculated constraints contribute to our knowledge of the eccentricity Sombor index and its possible applications in graph theory and other domains.

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On Optimal BV-control Problem in Coefficients for Elliptic Equation with Variable Order of Nonlinearity

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key-words: Dirichlet problem, p(x)-Laplace operator, optimal control, control in coefficients.

Abstract:
We deal with the following optimal control problem (OCP) in coefficients for nonlinear elliptic equation with nonstandard growth conditions (see [1] for more details)

$$\text{Minimize } I(u,y)=\int_\Omega |y-z_d|^2\,dx+\int_\Omega |Du|$$

subject to the constraints

$$-\operatorname{div}\left(u|\nabla y|^{p(x)-2}\nabla y\right)=-\operatorname{div} f \quad \text{in} \quad \Omega,$$

$$y=0 \text{ on } \partial\Omega,$$

$$u\in\mathfrak{A}_{ad}=\Big\{u\in BV(\Omega)\cap L^\infty(\Omega)\;\Big|\;\xi_1(x)\le u(x)\le\xi_2(x) \text{ a.e. in } \Omega\Big\},$$

$$2\le\alpha=\operatorname*{ess\,inf}_{x\in\Omega}p(x)\le p(x)\le\operatorname*{ess\,sup}_{x\in\Omega}p(x)=\beta<\infty, \quad \text{a.e. in } \Omega,$$

where Ω is a bounded open connected subset of $\mathbb{R}^N$ with a Lipschitz boundary $\partial\Omega$, the exponent $p:\Omega\to[2,\infty)$ is a Lebesgue-measurable function, the function $u\in BV(\Omega)\cap L^\infty(\Omega)$ is taken as a control, and $z_d\in L^2(\Omega)$ and $f\in L^\infty(\Omega;\mathbb{R}^N)$ are given distributions.

The characteristic feature of the considered OCP is the fact that the exponent $p(x)$ is assumed to be Lebesgue-measurable, and we do not impose any additional assumptions on $p(x)$ like to be a Lipschitz function or satisfy the so-called log-Hölder continuity condition.

In order to handle the inherent degeneracy of the $p(x)$-Laplacian, we use a special two-parametric regularization scheme. We derive existence and uniqueness of variational V-solutions to the underlying boundary value problem and the corresponding optimal control problem. We also discuss the asymptotic behaviour of the solutions to regularized problems on each (ε,k)-level as the parameters tend to zero and infinity, respectively.

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Korovkin Type Approximation Theorem via Statistical Relative Quasi Uniform Convergence

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key-words: Relative quasi uniform convergence, positive linear operator, Korovkin-type approximation theorem.

Abstract:
In this paper we present the notion of statistical relative quasi uniform convergence and using this type of convergence, we give a Korovkin-type approximation theorem. Finally, we give an example showing that our results are proper extensions of the corresponding classical ones.

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Convergence of Remote Projections onto Convex Sets

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key-words: Hilbert space, nearest point projection, convex set, convergence, symmetry.

Abstract:
Let $\{C_\alpha\}_{\alpha\in\Omega}$ be a family of closed and convex sets in a Hilbert space H, having a nonempty intersection C. We consider a sequence $\{x_n\}$ of remote projections onto them. This means, $x_0 \in H$, and x_{n+1} is the projection of x_n onto such a set $C_{\alpha(n)}$ that the ratio of the distances from x_n to this set and to any other set from the family is at least $t_n \in [0, 1]$. We study properties of the weakness parameters t_n and of the sets C_α which ensure the norm or weak convergence of the sequence $\{x_n\}$ to a point in C. We show that the condition

$$\forall\{a_\nu\} \in \ell_2 \text{ with } a_\nu \geq 0 : \liminf_{m\to\infty} \frac{a_m}{t_m} \sum_{\nu=1}^{m} a_\nu = 0 \tag{1}$$

is necessary and sufficient for the norm convergence of x_n to a point in C for any starting element and any family of closed, convex, and symmetric sets C_α. This generalizes a result of Temlyakov who introduced (1) in the context of greedy approximation theory. We give examples explaining to what extent the symmetry condition on the sets C_α can be dropped. Condition (1) is stronger than $\sum t_n^2 = \infty$ and weaker than $\sum t_n/n = \infty$. The condition $\sum t_n^2 = \infty$ turns out to be necessary and sufficient for the sequence $\{x_n\}$ to have a partial weak limit in C for any family of closed convex sets C_α and any starting element.

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Modified Exponential Sampling Series in Logarithmic Weighted Spaces of Functions

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key-words: Exponential sampling series, logaritmic weighted spaces of functions, modulus of continuity.

Abstract:
In this talk, we consider a modified form of the exponential sampling series constructed by Bardaro et al. [4] in 2017. In constructing the modified exponential sampling series, we consider the remaining part of the Mellin-Taylor formula based on the Mellin derivatives given in [5]. Then, we determine the approximation properties of the newly constructed series in logarithmic weighted spaces of functions. One can refer to [3], [1] and [2] as the first papers where the approximation properties of sampling series of exponential type in logarithmic weighted spaces of functions have been investigated.
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On the Generic Configurations of Two Dimensional Strongly Competing Systems

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key-words: Segregation of species, competition-diffusion system, Pattern formation, free boundary problem.

Abstract:
We study some qualitative properties of a well-known free boundary problem modelling strong segregation of N competing populations in planar domains. The main goal is to show that the domain generically divides into subdomains with only triple points, i.e. at most 3 populations meet at any point of the free boundary. To achieve this, we relate the solutions of the problem to a particular class of harmonic maps in singular spaces, which can be seen as the real part of certain holomorphic functions.

This is a joint work with E. Montefusco, V. Nesi and E. Spadaro (Mathematics Department, Sapienza University, Rome, Italy).

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Resolvent Approaches to Elliptic Regularity in Stationary Fokker-Planck Equations

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key-words: Stationary Fokker-Planck equations, Dirichlet forms, resolvents.

Abstract:
In this talk, we present the regularity results established in [1] for solutions to the stationary Fokker-Planck equation with general coefficients in dimensions $d \geq 3$. Specifically, we show that when the solution is interpreted as a measure with an L^2-density, this density not only belongs to the Sobolev space $W^{1,2}$ but also enjoys Hölder continuity. Our approach relies on constructing a reference measure $\mu = \rho dx$ through elliptic regularity and existence results, ensuring that the associated divergence-type operator fits within the framework of sectorial Dirichlet forms. By combining elliptic regularity results for both divergence-type and non-divergence-type equations, we establish $W^{2,2}$-regularity for the image of the resolvent operator. Furthermore, we show that the solution density arises as the weak limit of $W^{1,2}$-functions constructed via the resolvent, revealing the essential role of Dirichlet form techniques and resolvent approximations in understanding the regularity of stationary Fokker-Planck equations. We also anticipate that with refined estimates, our approach can extend to the two-dimensional case.

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Boundary Integral Equations of the First Kind: Solvability and Applications

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key-words: Boundary integral equations, potential theory, reducible operator, differential form.

Abstract:
In this talk, we focus on a method for solving boundary integral equations of the first kind, which naturally arise when seeking solutions to certain boundary value problems for elliptic differential equations in terms of single or double layer potentials. The resulting integral representations differ from those obtained through direct methods. One potential advantage of this alternative formulation is that the corresponding boundary integral equation may be more manageable, either numerically or analytically, depending on the specific context (see, e.g., [5]).

Coming back to the method used, it was introduced by Cialdea in [1] and it is based on the theory of reducible operators and the theory of differential forms.

Such a method has been extended to different boundary value problems associated with partial differential equations and systems, both in simply and multiple connected domains (see, e.g. [2,3,4] and the recent paper [6]).

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Existence and Monotonicity of Principal Eigenvalues for a Class of Fractional Boundary Value Problems

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key-words: Fractional boundary value problem, principal eigenvalue, fixed point theory.

Abstract:
In this talk, we investigate the spectral problem

$$\begin{cases}(-\mathcal{D}_{a+}^{\alpha}u)(x)=\lambda u, & x\in(a,b),\\ u(a)=u(b)=0, 1<\alpha<2,\end{cases}$$

where $\mathcal{D}_{a+}^{\alpha}$ represents the Riemann-Liouville fractional derivative. Using fixed point theory and positive operator theory, we establish the existence of principal eigenvalues for this problem. Interestingly, while the principal eigenvalue $\lambda(\alpha)$ is generally non-monotonic as a function of the order α, we show that the normalized quantity

$$\frac{\Gamma(\alpha-1)}{(b-a)^{\alpha-1}}\frac{1}{\lambda(\alpha)}$$

is strictly decreasing with respect to α.

If time permits, we will also discuss extensions to nonlinear cases of $f(u,x)$ and to a broader range of the fractional order parameter $1<\alpha<\infty$.

Design Construction and Model Selection for Small Mixture-Process Variable Experiments with High-dimensional Model Terms

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key-words: Bayesian, D-optimality criterion, generalized Bayesian-D criterion, split-plot design.

Abstract:
This paper considers the design construction and model selection for mixture-process variable experiments where the number of variables is large. For such experiments the generalized least squares estimates cannot be obtained and hence it will be difficult to identify the important model terms. To overcome these problems, here we employ the generalized Bayesian-D criterion to choose the optimal design and apply the Bayesian analysis method to select the best model. Two algorithms are developed to implement the proposed methods. A fish-patty experiment demonstrates how the Bayesian approach can be applied to a real experiment. Simulation studies show that the proposed method has a high power to identify important terms and well controls the type I error.

3rd International Conference: Constructive Mathematical Analysis

Sensitivity Approach for the Evaluation of Statistical Test Batteries

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key-words: FIPS 140-2, pseudo-random numbers, sensitivity, tests battery, tests suite.

Abstract:
For the verification of statistical properties such as randomness or uniformity, different hypothesis tests, generally non-parametric, are used to validate or not the generators that produced such sequences. In professional practice, in areas such as simulation, Monte Carlo techniques, applications in security and encrypted communication systems, etc., such verifications are performed by applying sets of tests called suites or batteries [1]. These batteries must be robust and contain uncorrelated tests to avoid possible interpretation biases and high computational times.
A current line of research is focused precisely on the analysis of the structure of batteries in order to make them as complete, robust and efficient as possible, see for example [2, 3, 4] or [6] among others.
In this paper we will discuss the main techniques used for battery analysis and also focus on the sensitivity of the FIPS 140-2 [5] tests battery.

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Fake Nodes Approach and Some Applications

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key-words: Mapped basis, fake nodes approach.

Abstract:
The concept of mapped bases has been widely studied, but all the proposed methods show convergence provided that the function is resampled at the mapped nodes. In applications, this is often physically unfeasible. We discuss the extention of the so-called *mapped bases without resampling* interpolation, also known as *Fake Nodes Approach* (FNA) [1, 2], to *any* basis and dimension. The univariate case has been discussed in [1] and some of its applications have been collected in the recent paper [2]. In the talk we show some relevant applications of this "fancy" approach.

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3rd International Conference: Constructive Mathematical Analysis

Rational Approximations of Fractional Elliptic Operators

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key-words: Fractional elliptic operator, rational approximation, exponential convergence.

Abstract:
Let $\mathcal{A}$ be a second-order elliptic differential operator in a bounded multidimensional domain Ω with general geometry. We consider the spectral definition of $\mathcal{A}^\alpha$. Over the past decade, several approaches have been proposed to approximate $\mathcal{A}^{-\alpha}$, $\alpha \in (0,1)$. Despite their different origin, they can all be written as a rational approximation.

We first discuss the BURA method introduced in [2] (see also the survey [1]). It is defined in the form $\mathcal{A}^{-\alpha} \approx \lambda_1^{-\alpha} r_{\alpha,k}(\lambda_1 \mathcal{A}^{-1})$. Here $r_{\alpha,k}(z)$ is the best uniform rational approximation of degree k to z_1^α, $z \in [0,1]$ and $\lambda_1 > 0$ is the minimal eigenvalue of $\mathcal{A}$. We demonstrate how the additive representation of the BURA approximation is a useful tool for qualitative analysis of $\mathcal{A}^\alpha$. We also show that the error decreases exponentially as k increases.

Let now the symmetric and positive definite matrix A be a mesh approximation of $\mathcal{A}$. A^α is dense, which is consistent with the nonlocality of $\mathcal{A}^\alpha$. The computational efficiency of the BURA method is due to the representation $A^{-\alpha} \approx \tilde{c}_0 (I + \sum_{i=1}^k \tilde{c}_i (A - \tilde{d}_i I)^{-1}$, where $\tilde{c}_i \geq 0$ and $\tilde{d}_i < 0$. Near-optimal estimates of computational complexity are obtained that do not depend on the condition number of A.

The last part of the talk is devoted to generalizations regarding the power α. The presentation follows and further develops the results of [3]. First, the advantages and disadvantages of the BURA and BURA-based methods for $\alpha > 0$ are discussed. Finally, the case $\alpha < 0$ is considered. It turns out that it is the most complex. Thus, approximating the fractional elliptic operator is more difficult than approximating its inverse. Accordingly, in the discrete case, developing a numerical method for multiplying a vector by A^α is a more difficult problem than solving a linear system with the same matrix.

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On the Complete Controllability of Fractional-Order Integrodifferential System Involving Generalized Conformable Derivatives

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key-words: Integrodifferential system, generalized conformable derivatives, semigroup theory, contraction mapping principle, complete controllability.

Abstract:
Controllability concepts have been essential across various disciplines, including engineering, robotics, optimal control, and applied mathematics. Following Kalman's definition, controllability is characterized by the ability to steer the solution of a dynamical system from any initial state to a desired target at a given finite time. In this paper, the complete controllability of fractional integrodifferential systems involving generalized conformable derivatives is investigated. A set of sufficient conditions for the complete controllability of Fractional-Order Integrodifferential semilinear systems is established, assuming that the associated linear part is completely controllable. The obtained results based on employing advanced mathematical tools, including the semigroup theory, fractional calculus, and the contraction mapping principle. A concrete example is provided at the end to illustrate the practical effectiveness of the theoretical findings.
Throughout this paper, the following fractional nonlinear system will be considered:

$$\begin{cases} {}^{GC}D^{p}z\left(t\right)=Az\left(t\right)+Bv\left(t\right)+h\left(t,z\left(t\right),\int_{0}^{t}g\left(t,r,z\left(r\right)\right)dr\right) & t\in]0,a] \\ z\left(0\right)=z_{0}\in Z, \end{cases} \tag{1}$$

Z is a Hilbert space, ${}^{GC}D^{p}$ is the generalized conformable derivative of order $0<p<1$, $A:Z\rightarrow Z$ is the infinitesimal generator of a strongly continuous semi-group $\{S(t),t>0\}$, $B:V\rightarrow Z$ is a bounded linear operator, $h:[0,a]\times Z\times Z\rightarrow Z, g:[0,a]\times[0,a]\times Z\rightarrow Z$ are non-linear continuous functions with appropriate conditions.
Here, some selected references:

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On Some Results Related to Positive Definite Matrices by Means of Cauchy-Schwarz Inequality

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key-words: Cauchy-Schwarz inequality, singular values, majorization, unitarily invariant norms.

Abstract:
In this study, the relationship between the Cauchy-Schwarz inequality and matrix norms and singular values is discussed. First, the classical form of the Cauchy-Schwarz inequality is recalled, and its extended versions in matrix space are examined. Then, the connections between singular values and unitarily invariant norms are addressed, and various inequalities are derived in this context. In particular, the majorization properties of singular values for Hermitian and positive semi-definite matrices are investigated. In addition, using some special matrix functions like Lieb functions and matrix convex functions, we get some inequalities for singular values and norms of positive semidefinite matrices.

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A New Generalization of the Frank Matrix

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key-words: Binomial matrices, max matrix, Frank matrix.

Abstract:
Matrices play a fundamental role in various areas of mathematics, including combinatorics, numerical analysis, and linear algebra. The literature includes significant papers on matrices whose entries are binomial coefficients, such as Pascal matrices. Max matrices, whose elements are defined as the maximum of their respective row and column indices, are other special matrices in matrix theory. Among them, the Frank matrix has recently attracted significant attention from researchers. Several studies have examined various generalizations of the Frank matrix and analysed their basic properties. This paper introduces a new generalization of the Frank matrix, called the binomial Frank matrix, and investigates some of its properties, including the determinant, inverse, adjoint matrix, LU decomposition, and certain norms. This study contributes to the literature by offering a novel approach to extending Frank matrices and providing insights into their algebraic and analytical properties.

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Highly Efficient Computational Technique for Equal Width Equation Modelling Dispersive Waves

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key-words: Collocation method, trigonometric B-splines, equal width equation, solitary waves.

Abstract:
The Equal Width (EW) equation, introduced by Morrison et al. [1], is given by

$$w_t + ww_x - \mu w_{xxt} = 0 \tag{1}$$

as a model for nonlinear dispersive waves. Recently, numerical solutions of the EW equation have gained popularity due to their simplicity compared to analytical solutions, which are limited to specific initial and boundary conditions. In this study, we aim to obtain an improved numerical solution by employing a fourth-order two-step scheme for temporal discretization and quartic trigonometric B-splines for spatial discretization. To validate the approach, the propagation of a single solitary wave is simulated. The accuracy of the proposed method is assessed by computing the L_∞ error norm and comparing it with results from the literature. Moreover, the temporal order of convergence and three conserved quantities are numerically evaluated and found to be consistent with their theoretical values. The results indicate that our method offers an efficient alternative compared to recent approaches.

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On Close to Convexity of a Class of Bounded Harmonic Mappings

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key-words: Convex, close-to-convex, harmonic.

Abstract:
The present study introduces a new subclass of bounded harmonic mappings. This subclass will represent existing concepts in the field and aims to provide new insights and potential applications to study harmonic functions. The purpose of defining these mappings is twofold: firstly, to deepen the understanding of their properties, and secondly, to broaden the scope of their usage in various mathematical contexts. This analysis investigates the conditions necessary for certain mappings to exhibit properties that make them approach close to be convex. Furthermore, sharp coefficient estimates are determined.
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Solving Sequential Linear Fractional Differential Equations Using α -Trigonometric Functions

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key-words: Fractional differential equations, Riemann–Liouville operators, α-trigonometric functions, mittag-leffler functions, oscillatory solutions, memory effects.

Abstract:
This paper introduces a novel analytical framework for solving sequential linear fractional differential equations (SLFDEs) with constant coefficients using α-trigonometric functions. These functions extend classical trigonometric forms via generalized Mittag-Leffler series to incorporate power-law memory and oscillatory dynamics. The approach enables the explicit construction of both homogeneous and non-homogeneous solutions, preserving non-local memory effects intrinsic to fractional-order systems. A new concept the α-Wronskian is defined to assess the linear independence of solutions, paralleling the classical Wronskian in ordinary differential equations. Applications to fractional harmonic oscillators illustrate the framework's capability in capturing damped and amplified responses driven by memory kernels. The proposed method significantly enhances the modeling of complex physical and engineering systems governed by fractional dynamics.

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Generalized Gamma-Type Operators

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key-words: Generalized Gamma distribution, approximation by positive linear operators, asymptotic formula, rate of convergence.

Abstract:
In this talk we present a class of integral operators of probabilistic type, recently introduced in [1], which are constructed by means of the generalized Gamma distribution. In particular, we investigate their approximation properties in weighted continuous functions spaces and L^p-spaces, providing some estimates of the rate of convergence by means of different moduli of smoothness (such as, e.g., the ones introduced in [2]). An asymptotic formula is also discussed.

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The Probabilistic Characteristics of Ion-Acoustic Waves in a Randomized Plasma Model with Non-extensive Electrons

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key-words: Plasmas with uncertain properties, random variable transformation, probability density function, mean, variance, confidence interval.

Abstract:
In this study, a randomized plasma model consisting of positive ions and non-extensive electrons is presented. The uncertain properties of the plasma system are treated as continuous random variables. A probabilistic charac- terization of ion-acoustic wave propagation is then analyzed by calculating the first probability density functions, along with the corresponding means, variances, and confidence intervals. To achieve this, the random variable transformation (RVT) method [1, 2, 3] is applied as a probabilistic approach. The theoretical findings are validated through numerical simulations, which show strong agreement with physical expectations.

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Propagation of Solitons and Nonlinear Behavior in Nonlinear Power Law Fibers

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key-words: The complex Ginzburg–Landau equation, soliton solutions, the φ^6-model expansion technique.

Abstract:
In this work, we study the soliton propagation in the context of a detuning factor in nonlinear optics, represented by the complex Ginzburg–Landau equation (CGLE). The φ^6-model expansion technique is used to explicitly obtain various solutions, including trigonometric, hyperbolic, and rational function solutions. Additionally, singular soliton solutions are seen as positive. Using nonlinear power law fibers, the model is examined. The findings of this work are expected to contribute to the enhancement of the nonlinear dynamical properties of the CGLE. The approach suggests a useful and efficient technique for getting reliable solutions to a large number of nonlinear partial differential equations. Some of the results that were discovered are shown in contour, two-dimensional, and three-dimensional plots.

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A Constructive Definition of the Riemann Integral on a Separable Banach Space

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key-words: Riemann integral, Lebesgue integral, separable Banach space.

Abstract:
The goal of this talk is to construct a Riemann integral on a separable Banach space which possesses all of the fundamental properties of the Riemann integral on $\mathbf{R}^n$. Let $\mathcal{B}$ represent a separable Banach space. The paper [1] presents a proof that $\mathcal{B}$ has an isomorphic, isometric embedding in $\mathbf{R}^\infty$. In this work we will use this embedding to define a Riemann integral on special subsets of $\mathcal{B}$, which makes the derivations of most of its properties virtually identical to those of its finite-dimensional analogue. Similar to the Lebesgue integral on $\mathcal{B}$, this Riemann integral has the advantage of equaling a limit of Riemann integrals on $\mathbf{R}^n$ as $n \to \infty$.

We will use this convergence to study some probability density functions on $\mathcal{B}$.

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Geometry of Curves as a way Establish Connections between Integrable Nonlinear Equations

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key-words: Spin systems, integrable nonlinear equations, geometry of curves.

Abstract:
We have investigated the spin equation with self-consistent potentials of the form [1]:

$$S_t + S_{xxt} + (vSS_x)_x + \frac{1}{4}(\{S_x, S_t\}S)_x - (wS)_x = 0, \tag{1}$$

$$w_x + \frac{1}{4}(tr(S_x^2))_t = 0, \tag{2}$$

$$v_x + \frac{1}{4}tr(S[S_x, S_t]) = 0, \tag{3}$$

here

$$S = \begin{pmatrix} S_3 & S^- \\ S^+ & -S_3 \end{pmatrix}, \quad S^{\pm} = S_1 \pm iS_2, \quad S^2 = \Sigma, \quad \Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

It has been established that equation (1) and equation [1]:

$$q_t + q_{xxt} - vq - (wq)_x = 0, \tag{4}$$

$$r_t + r_{xxt} + vr - (wr)_x = 0, \tag{5}$$

$$v_x + 2(r_{xt}q - rq_{xt}) = 0, \tag{6}$$

$$w_x - 2(rq)_t = 0, \tag{7}$$

where $q(x,t)$ and $r(x,t)$ are complex functions.

are equivalent to each other by means of the geometry of curves and surfaces. Exact soliton solutions of equation (1)-(1) have been found, and the same solutions of equation (4)-(7) have been constructed through that geometric equivalence [2]-[3].

The approach proposed in this study makes it possible to clarify the nature of the close connection between integrable nonlinear partial differential equations through the differential geometry of curves and surfaces.

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Zero Divisors of a Pythagorean Ring

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key-words: Pythagorean ring, zero divisors, quotient ring.

Abstract:
Let P denote the ring of the Pythagorean triples and $Z(P)$ is its subset of zero divisors. It is well known that P is isomorphic to Z^2. Even though it is straight forward to identify the zero divisors in Z^2, the identification of zero divisors in P is quite laborious. Since $Z(Z^2)$ is not an ideal of Z^2, the isomorphism between the two rings implies that $Z(P)$ is not an ideal of P. However, some subsets of $Z(P)$ form ideals of P. In this paper, we have answered the following question: Given that $\varphi : P \to Z^2$ is an isomorphism, what is $\varphi^{-1}\left(Z\left(Z^2\right)\right)$? The zero divisors in P have been identified and partitioned into ideal forming subsets and consequently constructing, new (quotient) rings P/I and I being ideal forming subset of $Z(P)$. Further, some interesting results on the zero divisor graph of P are obtained.

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An Alternative Method for Associated Curves in Modified Orthogonal Frame

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key-words: Associated curves, modified orthogonal frame.

Abstract:
This study defines tangent, principal normal and binormal associated curves with modified orthogonal frame in Euclidean 3-space so that each of these curve's vector is in its partner's osculator, normal and rectifiying plane, respectively.A new moving frame and its curvatures are generated with each associated curve. In addition the solutions of the distance funcitons between the curve and its associated curve are investigated and possible results were examined seperately for different curves.It is obtained that involute curves belong to the tangent associated curved in general and Bertrand and Manheimm curves belong to the principal normal and binormal associated curves. Finally we take some curves and their associated curves as examples and show them graphically.

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Fixed Point Theorems in Extended G-Menger Spaces and Application

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Abstract:
This paper introduces the concept of extended G-Menger spaces, a novel framework designed to unify and generalize several contemporary metric structures. Our construction provides a comprehensive extension of G-Menger spaces, Menger probabilistic metric spaces, controlled strong b-Menger spaces, and generalized G_b-metric space, offering a more versatile setting for analysis under uncertainty. The core of our investigation lies in establishing new fixed-point theorems for probabilistic φ-contraction mappings within this generalized context. By leveraging the properties of continuous t-norms in conjunction with a newly introduced bounded control function, we rigorously demonstrate the existence and uniqueness of fixed points under well-defined conditions. Beyond the theoretical contributions, we investigate the fundamental topological properties of these spaces. To showcase the practical power and applicability of our framework, we present a direct application to solving a Volterra-type integral equation, thereby bridging abstract theory with a concrete problem in functional analysis. The results presented herein not only subsume and extend a range of existing findings but also contribute a robust new tool to the broader fields of fixed-point theory and probabilistic analysis.

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Fixed Point Theory in Graphical Modular Metric Spaces: A Unified Approach via Banach Kannan, and Reich-Type Contractions

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key-words: Graphical modular metric space, graphical modular-Banach contraction, graphical modular-Kannan contraction, graphical modular-Reich contraction.

Abstract:
This study introduces and develops the novel framework of graphical modular metric spaces, an innovative synthesis of modular metric theory and graph-based structures. Motivated by recent advances in fixed point theory and the emerging utility of graph-theoretic metrics, we define and analyze non-Archimedean graphical modular metric spaces, extending classical concepts in a structured and topologically enriched setting. Within this framework, we investigate the existence and uniqueness of fixed points for three prominent contraction types- Banach, Kannan, and Reich contractions—in the context of directed graph-preserving mappings, in which the obtained results generalize classical fixed point theorems and provide a unified theoretical foundation for fixed point analysis in modular metric spaces endowed with graph connectivity.

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Convergence of a Family of Mellin-Steklov Type Exponential Sampling Operators

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key-words: L^p convergence, Mellin-Steklov integrals, exponential-type sampling series.

Abstract:
In this talk, we introduce Mellin-Steklov exponential sampling operators of order r, where $r \in \mathbf{N}$, defined through appropriate Mellin-Steklov integrals. We investigate the approximation properties of these operators in the spaces of continuous and bounded functions and L^p spaces $(1 \leq p < \infty)$ on $\mathbf{R}^+$. Additionally, we present a quantitative Voronovskaja- type theorem and examine the convergence behavior of these newly defined operators in logarithmically weighted function spaces.
Acknowledgements: The first author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The author thanks to TUBITAK for their supports.

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A New Generalization of the Digital Contraction Principle and Image Compression Applications

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key-words: Digital contraction, digital metric space, image compression.

Abstract:
In 2000 Branciari [1], defined generalized metric spaces by modifying the triangle inequality condition of metric spaces in the classical meaning with the condition

$$d(x,y) \leq d(x,u) + d(u,v) + d(v,y)$$

for every $x, y \in X$ and different points $u, v \in X$ (each different from x and y). Then in 2014, Jleli and Samet [2], extended the Banach contraction principle by defining a class of $\Theta-$functions on generalized metric spaces. In 2015, Ege and Karaca [3], introduced a new perspective to the literature by defining digital metric spaces and proving Banach's fixed point theorem on these spaces. In this talk, a fixed point result will be given using the $\Theta-$function class in generalised digital metric spaces. Also various image compression applications will be discussed.
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Convergence Properties for a New Version of the Mellin-Gauss-Weierstrass Operators

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key-words: Convergence, Mellin-Gauss-Weierstrass operators, Voronovskaya-type theorem.

Abstract:
In this presentation, we give the convergence properties for a new version of the Mellin-Gauss-Weierstrass operators. Furthermore, a Voronovskaya-type theorem is expressed. This expression includes Mellin derivatives and the logarithmic moment.
Acknowledgements: The author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The author thanks to TUBITAK for their supports.

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Modular Fixed Point Framework for Seljuk Knot Motifs: A Topological Study of Karatay Madrasa and Kubadabad Palace

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key-words: Kubadabad Palace, Karatay Madrasa, fixed point theorem, knot theory.

Abstract:
This study proposes a novel integration of fixed point theory and Seljuk architectural design by modelling geometric tile motifs using modular function spaces. Focusing on knot-based decorations from the Karatay Madrasa and Kubadabad Palace in Konya, we explore the applicability of modular topology in capturing the recursive and invariant nature of these motifs. Using various types of contractive mappings, we show the existence of fixed points corresponding to periodic and symmetric tile patterns. A comparative analysis between metric and modular spaces is provided to justify the use of modular functionals in modelling geometric features such as rotation invariance and curve embeddings. This interdisciplinary approach bridges cultural heritage and mathematical rigor, paving the way for digital reconstruction and pattern recognition in historical architecture.

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3rd International Conference: Constructive Mathematical Analysis

On Parabolic Class Number of a Special Subgroup of the Modular Group

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key-words: Congruence subgroups, parabolic class number, Fuchsian groups.

Abstract:

In this study, we investigate a constant of the signature of a special subgroup G_p of the modular group Γ, defined as

$$G_p = \left\{ \begin{pmatrix} a & b \\ cp^\alpha & d \end{pmatrix} \in \Gamma : a \equiv d \pmod{p} \right\}$$

where p is a prime number and α is any natural number. The main focus is the determination of the number of parabolic conjugacy classes, which correspond to the cusps of the modular surface associated with this group. Using group-theoretic and arithmetic methods, we explicitly compute the parabolic class number of G_p. The results contribute to a deeper understanding of the structural and geometric properties of modular subgroups beyond the classical congruence cases.

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A New Proof of the Girard-Fermat Theorem

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key-words: Prime numbers, Fermat, two squares theorem, congruence equations.

Abstract:
In this paper, we provide a novel and straightforward proof of the well-known Girard–Fermat Theorem, which asserts that every prime $p \equiv 1 \pmod 4$ can be written as the sum of two squares. By utilizing a result derived from Wilson's Theorem, we offer an elegant and simplified proof, which contrasts with more complex approaches traditionally used to establish this result in number theory.

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Mappings Contracting Triangles

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key-words: Metric space, fixed point theorem, three-valued mappings, contracting triangles.

Abstract:
The aim of the current paper is to introduce a new class of contractive mappings, which are contracting (a feature of) triangles and prove the fixed point result for such mappings. We emphasize that our main theorem encompasses many functions for which the result holds, thus being a significant improvement in the research field.

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Properties of Hyperbolic Convolution on the Unit Disc

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key-words: Hyperbolic convolution, approximate identity on the unit disc.

Abstract:
The Blaschke group was introduced by Pap and Schipp in [1, 2]. Using the representations of the Blaschke group, it was possible to generate hyperbolic wavelet transforms and multiresolution in the analytic function spaces (see [3, 4, 5, 6, 7, 8]). A subgroup of the Blaschke group induces group operation on the unit disc. Recently, Schipp introduced a new convolution operator in this group [9]. In this paper, we study further properties of the hyperbolic convolution over the unit disc, and we derive approximation identity results on the unit disc.

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Optimal Recovery of Mappings Based on Linear Information with the help of T–splines in Banach Spaces

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key-words: Optimal recovery, information operators, abstract splines.

Abstract:
Let H, X, Y be real Banach spaces, $\|\cdot\|_X$ be a norm in X, M be some subset of H, $\mathcal{L}(X,Y)$ be the space of linear bounded operators $A\colon X \to Y$, X^* be the space dual to X, A^* be the adjoint operator for A. By $\mathcal{R}(A)$ we denote the range of the operator A, and by $\mathcal{N}(A)$ we denote the kernel of A. The values of the functional $f \in X^*$ on $x \in X$ we denote as follows $\langle x, f\rangle_X$. For a given operator $A \in \mathcal{L}(X,Y)$ we set $\mathcal{N}(A)^\perp = \{f \in X^*\colon \langle x, f\rangle_X = 0 \quad \forall x \in \mathcal{N}(A)\}$. As it is well known, $\mathcal{N}(A)^\perp = \overline{\mathcal{R}(A^*)}$, where $\overline{Q}$ is the closure of the set Q.

We define the best approximation of a class $A(M)$ by a subspace F in space X by the following

$$E(A(M); F; X) := \sup_{x \in A(M)} \inf_{u \in F} \|x - u\|_X.$$

Set

$$P_F x = \{u \in F\colon \|x - u\|_X = E(x; F; X)\}$$

is called a metric projection of element x onto the set F. If for any $x \in X$ set $P_F x$ contains exactly one element, then the set F is called a Chebyshev set.

Let $T \in \mathcal{L}(H,Y)$ be an information operator. Elements of the manifold

$$S(T) = S_H(T) := \{s_h = h - P_{\mathcal{N}(T)}h\colon h \in H\}$$

are called T-splines in the space H.

We say that T-spline $s_h \in S(T)$ is T-interpolating for an element $h \in H$ if $Ts_h = Th$.

We prove that for an arbitrary $h \in H$ there exists a unique T-interpolating spline s_h. We also obtain a representation of such a spline via Th and establish a minimization property of T-interpolating splines. Suggested definition of $E(A(M); F; X)$ is aimed at studying extremal problems of approximation theory on classes $A(M)$ of elements of a Banach space X, given as an image of a set $M \subset H$ under mapping $A\colon H \to X$ (not necessarily linear). Many important classes of periodic and nonperiodic univariate and multivariate functions, in particular classes of solutions to various problems of mathematical physics, can be represented in such a way.

We also use T-interpolating splines to solve the problem of optimal recovery of classes $A(M)$ in space X based on available information on elements of set M, given with the help of information operator T.

Some Fixed Point Theorems in Modular b-Metric-Like Spaces

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key-words: b-metric-like spaces, modular b-metric-like spaces , fixed point theory, contraction principle.

Abstract:
The aim of this paper is to contribute to the fixed point theory by proving some fixed point theorems in modular b-metric-like space. The existence and uniqueness of fixed point in the modular b-metric-like space under different contraction conditions are handled.

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Recent Advancements on Semi-Discrete Sampling Operators in Sobolev-Orlicz Spaces

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key-words: Durrmeyer sampling operators, simultaneous approximation, Sobolev-Orlicz spaces, maximal operators.

Abstract:

In this talk, we present some recent approximation results for a semi-discrete family of sampling operators, called Durrmeyer Sampling Operators (DSO) [3], in Sobolev-Orlicz spaces [4]. This research is part of the theory of sampling series, which plays a key role in Approximation Theory and its applications, especially in Signal and Image Processing. DSO extend classical families such as generalized [2] and Kantorovich [1] sampling operators, being based on two different kernels: a discrete and a continuous one.

We present new convergence results for Durrmeyer-type sampling operators (DSO) in Sobolev–Orlicz spaces, focusing on the simultaneous approximation of functions and their weak derivatives [6]. These results are based on a density method and on the boundedness of maximal operators in Orlicz spaces [7, 5].

We also study a direct approach involving φ-functions that do not satisfy the Δ_2-condition. This leads to convergence results with order of approximation in more general functional spaces, both in the sense of the Luxemburg norm and modular convergence.

Several examples are discussed, including Sobolev–Orlicz versions of Zygmund and exponential-type spaces, as well as classical Sobolev spaces.

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A Study of Fixed Points in Soft S-Metric Spaces via Hybrid-Interpolative Reich-Istrecescu Type Mappings

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Abstract:
Classical fixed point theory has long been one of the foundational pillars of mathematical analysis and topology, with significant developments emerging over the past century. Early groundbreaking contributions include Brouwer's Fixed Point Theorem (1911) and Banach's Contraction Principle (1922), which have played a critical role in shaping both theoretical and applied mathematics. In particular, Banach's theorem guarantees the existence and uniqueness of fixed points for contraction mappings on complete metric spaces, providing a robust framework widely applicable in nonlinear analysis, differential equations, and numerical methods.

Over time, the limitations of classical fixed point theorems within standard metric spaces have become apparent, motivating the development of generalized metric structures. As a result, new concepts such as G-metric, b-metric, and S-metric spaces have been introduced. Among these, S-metric spaces, initially defined by Sedghi et al.[5], extend the classical notion of metric spaces by utilizing a three-variable distance function, thereby offering a more flexible analytical environment. These spaces have opened new avenues for exploring fixed point results under weaker or more generalized conditions.

Currently, three primary approaches are predominant in the generalization of fixed point theory:

- generalizing the contraction conditions,
- extending the underlying metric structure, and
- investigating fixed points in terms of geometric constructs.

This study employs a combination of the first two approaches by both introducing a new contraction framework and working within a broader metric structure, namely soft S-metric spaces.

Soft set theory, proposed by Molodtsov in 1999 [4], emerged as a powerful mathematical tool for handling uncertainty and imprecise information. When integrated with metric concepts, it has led to the development of soft metric spaces, and more recently, soft S-metric spaces, which offer an enriched setting for analyzing fixed point problems in uncertain environments [1, 2]. These structures are particularly valuable in fields such as decision-making, artificial intelligence, and systems analysis.

In this study, we adopt the hybrid-interpolative Reich-Istrecescu type contraction, introduced by Karap1nar et al. [3], and adapt it to the context of soft S-metric spaces. This generalized contraction framework, characterized by a hybrid and parameterized structure, broadens the classical Reich-Istrecescu model and facilitates the derivation of new fixed point theorems. Within this extended setting, we establish both existence and uniqueness results, further supported by illustrative examples.

Therefore, this work contributes not only to the generalization of fixed point results but also demonstrates the effective integration of soft structures into the realm of metric fixed point theory. In doing so, it expands the theoretical scope and application potential of fixed point theorems in more complex and uncertain systems.

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Normal Structure and Contractions which Diminish Radius in Metric and Banach Spaces

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key-words: Metric space, admissible subset, hyperconvex metric space, mappings diminishing the radius, orbitally wrt nonexpansive mapping, orbitally Kannan mapping, weak normal structure, weak quasi-normal structure, convex invariant subset, fixed point.

Abstract:
in this talk, by using admissible sets, we give some fixed point results for orbitally contractions which diminish the radius of invariant convex subsets and orbits. Furthermore, a characterization of the weak normal structure by the fixed point property associated with this class of mappings is established. .

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Sampling Type Operators Versus AI in Medical Image Processing

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key-words: Sampling Kantorovich operators, segmentation algorithm, image processing, U-Net neural network.

Abstract:
In this talk we will delve into recent outcomes highlighted in [4], focusing on the application of sampling-type operators in digital image processing. A key role in this work is undertaken by sampling Kantorovich (SK) operators ([1, 4]), widely appreciated for their accuracy in the reconstruction of signals that are not necessarily continuous. Building upon significant theoretical findings, our study employs these operators within a deterministic method grounded on a segmentation algorithm, in which the SK assume a pivotal role ([2, 5]). This mathematically-based approach is compared with an AI method that involves a U-Net neural network. Both approaches are aimed at segmenting the patent area of the aortic vessel from CT images of patients affected by abdominal aortic aneurysm, proposing innovative and less invasive alternatives to nephrotoxic contrast agents commonly employed in diagnosing this pathology. The numerical and visual assessment of the outcomes highlights the efficiency and reliability of both methods.

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Respecting Cauchy Equivalence Classes

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key-words: Cauchy sequences, equivalence classes, continuity, removable discontinuities.

Abstract:
In the standard construction of real numbers as Cauchy equivalence classes of rational sequences, only the basic arithmetic operations are defined to respect these classes, leaving other functions prone to discontinuities or undefined points that necessitate limits. This reliance on limits reintroduces complexity into a system initially built on convergence, undermining the elegance of the Cauchy construction. This paper proposes a unified approach where all functions respect Cauchy equivalence classes, ensuring they are well-defined at all points without redundant limits. By applying rational functions to sequence representatives and assigning the resulting equivalence class, we eliminate pathologies such as removable discontinuities. Examples, including difference quotients and Riemann sums, demonstrate the method's efficacy. Our approach offers simplicity and coherence, suggesting a principled construction of real analysis.

Notation. By default, an unbracketed 1 is a rational number, and bracketed $[1]$ is real.

Method. A Cauchy sequence $(q_n)_{n\in\mathbb{N}}$ in $\mathbb{Q}$ satisfies $\forall\epsilon > 0, \exists N : m, n > N \implies |q_m - q_n| < \epsilon$. Two sequences (q_n) and (r_n) are equivalent if $\forall\epsilon > 0, \exists N : n > N \implies |q_n - r_n| < \epsilon$, and $\mathbb{R}$ is the quotient set.

For a rational partial function $f : \mathbb{Q}^k \to \mathbb{Q}$, we define its *Cauchy lift* ${}^c f : \mathbb{R}^k \to \mathbb{R}$ s.t. if $(x_1, \ldots, x_k) \in \mathbb{R}^k$ and $Y \in \mathbb{R}$ then ${}^c f(x_1, \ldots, x_k) = Y$ if for all $(q_{1,n}) \in x_1, \ldots, (q_{k,n}) \in x_k$, if $f(q_{1,n}, \ldots, q_{k,n})$ is defined, then $f(q_{1,n}, \ldots, q_{k,n}) \in Y$.

In other words, if f is undefined for some representatives, ${}^c f$ remains well-defined as long as the defined cases consistently yield sequences in Y. This leverages the equivalence class structure to extend f naturally across $\mathbb{R}$, avoiding artificial undefinedness.

Example 1. We construct ${}^c\sqrt{X}$ from the rational $\sqrt{x}$. For $X \in \mathbb{R}$, take $x = \{c_1, c_2, c_3, \ldots\} \in X$, and define $\sqrt{x} = \{\sqrt{c_1}, \sqrt{c_2}, \sqrt{c_3}, \ldots\}$ when each c_i is a rational perfect square. We require a $Y \in \mathbb{R}$ such that if $\sqrt{x}$ is defined, then $\sqrt{x} \in Y$. For $X = [2]$, take $x = \{1^2, 1.4^2, 1.41^2, 1.414^2, \ldots\}$, so $\sqrt{x} = \{1, 1.4, 1.41, 1.414, \ldots\} \in [\sqrt{2}]$. Similarly, sequences like $\{3, 3.9, 3.99, \ldots\} \in [4]$ may not have defined square roots pointwise, but whenever defined (e.g., perfect squares approximating 4), the result is in $[2]$. Thus, ${}^c\sqrt{X} = Y$, is well-defined for all $X \geq 0$.

Example 2. To find the derivative of $f(x) = x^2$ at $x = 1$, consider the rational function $g(h) = \frac{(1+h)^2 - 1^2}{h} = 2 + h$, defined for $h \neq 0$. Construct ${}^c g(H)$ for $H \in \mathbb{R}$. Take $h = \{c_1, c_2, c_3, \ldots\} \in H$ with all $c_i \neq 0$, so $g(h) = \{2 + c_1, 2 + c_2, 2 + c_3, \ldots\} \in [2 + H]$. For $H = 0$, use $h = \{\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \ldots\}$, yielding $\{2 + 1, 2 + \frac{1}{2}, 2 + \frac{1}{3}, \ldots\} \in [2]$. So, ${}^c g(H) = 2 + H$. Thus, ${}^c g(0) = 2$, the derivative $f'(1)$, directly obtained, without redundant limits.

Example 3. For $g(x) = x$ from 0 to 1, define the rational Riemann Sum function $f(h) = h \cdot \sum_{i=1}^{1/h} ih$. Construct ${}^c f(H)$ at $H = 0$ with $h = \{1, \frac{1}{2}, \frac{1}{3}, \ldots\} \in 0$, where $1/h_n = n$ is an integer. This simplifies $f(h) = h^2 \cdot \frac{\frac{1}{h}(\frac{1}{h}+1)}{2} = \frac{1+h}{2}$. Then, $f(h) = \{\frac{1+1}{2}, \frac{1+\frac{1}{2}}{2}, \frac{1+\frac{1}{3}}{2}, \ldots\} = \{1, \frac{3}{4}, \frac{2}{3}, \ldots\} \in [\frac{1}{2}]$. Thus, ${}^c f(0) = \frac{1}{2}$, the integral $\int_0^1 x\,dx$, directly obtained, again without redundant limits.

Summary. By mandating that all functions respect Cauchy equivalence classes, we redefine real analysis to be more coherent and intuitive, avoiding redundant limits and pathologies inherent in the classical approach. Examples confirm practicality for diverse functions. Future work could extend to other quotient constructions, potentially reinterpreting mathematical analysis.

S^*_{pmk} Index for the Burr X-Perks Distribution with Applications

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key-words: Statistical process control, Burr X–Perks distribution, maximum likelihood estimation, interval estimation, process capability index, symmetric and asymmetric loss function.

Abstract:
Process capability indices are widely employed in quality control practices to quantitatively assess the extent to which manufacturing processes meet customer requirements. These indices provide comprehensive information about a process's performance and quality level by evaluating whether the process outputs fall within predefined specification limits. Selecting an appropriate process capability index based on the distributional characteristics of the observed data is crucial for ensuring an accurate assessment of the process. This study proposes a novel process capability index that incorporates both symmetric and asymmetric loss functions. The properties of the proposed index are thoroughly examined under the assumption that the process data follow a Burr X–Perks distribution. The point and interval estimators of the proposed index are developed using the maximum likelihood methodology, and their performance is evaluated through an extensive Monte Carlo simulation study. Furthermore, a real data application is presented to demonstrate the practical applicability of the proposed index.
Acknowledgements: The first author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The author thanks to TUBITAK for their supports.

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Variable-Order Hadamard-Type Fractional Calculus with respect to Function

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key-words: Fractional Calculus, Ψ-Hadamard-type fractional integral of variable order (FIVO), Ψ-Hadamard-type fractional Derivative of variable order (FDVO), existence and uniqueness results.

Abstract:
In this paper, We introduce the Ψ-Hadamard-type fractional integral of variable order (FIVO). We propose two variants of the Ψ-Hadamard-type fractional derivative of variable order (FIVO) in the Riemann–Liouville sense and three variants in the Caputo sense. We study their relevant properties, established their interconnections and rigorously prove the relationship between these operators. In addition, We drive existence and uniqueness results for fractional differential equations involving the Riemann–Liouville Ψ-Hadamard-type fractional derivative operator of constant order by applying fixed-point theorems. Finally, we illustrate the theorem with examples.

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Adjoint Curve Using Conformable Derivative

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key-words: Conformable derivative, adjoint curve, Euclidean space, curvatures.

Abstract:
This study examines the geometric properties of curves in three-dimensional Euclidean space by conformable derivatives, which provide an improved correspondence to classical differentiation compared to conventional fractional derivatives. The emphasis is on the adjoint curve, which undergoes significant alterations when examined via conformable derivatives. We describe the conformable adjoint curve and examine its construction, the interrelations among curvatures, and the Frenet frame vectors. Furthermore, we examine the Darboux vector and the criteria under which the adjoint curve constitutes a general helix. We illustrate, via theoretical analysis and graphical representations, the influence of conformable derivatives on the shape of the adjoint curve.

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3rd International Conference: Constructive Mathematical Analysis

Structural Modeling of Covariate-Driven Misclassification Risk: A Likelihood-Based Approach

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key-words: Covariate effects, likelihood-based estimation, logit model, measurement error, misclassification probability.

Abstract:
This study presents a likelihood-based estimation framework developed to analyze how the risk of misclassification in quality control settings varies with measurement-related covariates. In industrial inspection systems, items are classified as conforming or nonconforming based on measured characteristics that are often subject to noise. When the precision of these measurements changes across operators, machines, or environmental conditions, the probability of incorrect classification becomes condition-dependent. The proposed model expresses this misclassification probability as a logistic function of covariates and incorporates it into the overall variance structure of the observed measurements via a numerically constructed inverse mapping. Simulation results demonstrate that the proposed method successfully recovers the covariate effects that drive misclassification and yields consistent parameter estimates under varying sample sizes. This approach provides a flexible and interpretable tool for identifying measurement conditions that impact classification reliability in quality assurance processes.
Acknowledgements: The first author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The author thanks to TUBITAK for their supports.

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An Inverse Problem for Pseudoparabolic Equation with Memory Term and Damping

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key-words: Inverse problem, nonlinear pseudoparabolic equation, memory term, solvability.

Abstract:

$$u_t - \kappa\Delta u_t - \lambda\Delta u - \int_0^t K(t-s)\Delta u(x,s)ds = \gamma\,|u|^{q-2}\,u + f(t)\cdot g(x,t),\ \text{ in } Q_T, \tag{1}$$

the initial condition

$$u(x,0) = u_0(x) \text{ in } \Omega \tag{2}$$

the boundary condition

$$u(x,t) = 0 \text{ on } \Gamma_T, \tag{3}$$

and the integral overdetermination condition

$$\int_\Omega u(x,t)\omega(x)dx = h(t),\ \ t \in [0,T]. \tag{4}$$

Here, the coefficient κ, λ are given positive numbers, γ is the coefficient of the damping term might be positive $\gamma > 0$ either negative $\gamma < 0$. The functions $g(x,t),\ u_0(x),\ \omega(x)$ and $h(t)$ are given. The exponent q is given positive number such that

$$1 < q < \infty. \tag{5}$$

The main goal of this work is to show existence and uniqueness of weak solutions to the inverse problem (1)-(4).

An Insight into Effects of Geography and Topography on the Urban Roadway Network Development of the Istanbul through Fractal Analysis

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key-words: Fractal analysis, urban morphology, Istanbul, urban network analysis.

Abstract:
Urban roadway networks are complex systems shaped by various geographical and topographical factors. These elements influence not only the physical layout of roads but also their connectivity, accessibility, and overall efficiency. Fractal analysis, a mathematical approach to assess complex structures, has emerged as a valuable tool for understanding these networks' configurations and growth patterns. This research investigates and provides an insight through fractal analyses into how geography and topography can impact the development of urban roadway networks. Istanbul, which is a major metropolitan world city is composed of 39 districts. The study focuses on three of these districts that form the historical triad of Istanbul: Fatih, Beyoglu, and Uskudar. Separated by a unique waterway network composed of the Golden Horn, the Bosphorus Strait and the Marmara Sea, Istanbul's easterly, westerly and northernly expansion emanated from this peculiar district triad that form the historical core of Istanbul's transportation network.

The study uses Geographic Information System (GIS) tools to extract the roadway networks of the studied districts and Benoit Software to conduct fractal analyses on the roadway networks to calculate and compare fractal dimensions estimated by five different techniques referred to as 1. Box Counting (Db), 2. Perimeter Area (Dp), 3. Information (Di), 4. Mass (Dm), and 5. Ruler (Dr). The analysis will initially focus on the development of roadway networks within these three districts and subsequently extend the examination towards west from Fatih District, toward North from Beyoglu District and towards east from the Uskudar District.

This paper provides an insight into how fractal analysis can offer a strong method to represent the development of urban roadway networks in relation to urban topography and urban geography. Istanbul, as the subject of this paper, provides a valuable case study by consideration of its multitude of geographical features that are unique to this city. The study will discover ways to use fractal analyses to define connectivity and accessibility offered by urban roadway networks and discuss how such mathematical means of network representation can lead us to make informed choices in urban roadway network development.

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Controlling Periodic Limit Cycles of Math Anxiety

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key-words: Limit cycles, math anxiety, epidemiological modeling.

Abstract:
Math anxiety among college students often displays cyclical fluctuations, particularly around assessment periods. In this study, we modeled the temporal dynamics of math anxiety using a compartmental framework adapted from mathematical epidemiology. By fitting this model to longitudinal data on students' self-reported anxiety levels, we characterized the emergence of periodic oscillations and identified key parameters influencing their amplitude and persistence. We then simulated targeted interventions such as supplemental academic support delivered prior to examinations to assess their impact on reducing anxiety peaks. Additionally, we derived the minimal intervention threshold necessary to disrupt sustained anxiety cycles. Given the socially transmissible nature of math anxiety, our findings suggest that strategies grounded in infectious disease modeling can inform effective approaches for its mitigation in educational settings.

Julia Sets-Geometric and Potential-Theoretic Insights

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key-words: Filled Julia sets, iterations, discs, Fatou sets, non-autonomous Julia sets, Green function.

Abstract:
We present some results concerning closed discs contained in the filled Julia sets associated with polynomials $f_c : \mathbb{C} \ni z \longmapsto z^2 + c \in \mathbb{C}$ mostly for $c \in \left[-2, \frac{1}{4}\right]$ but also some other values. We investigate also a few non-autonomous Julia sets. As an application we mention that the pluricomplex Green function associated with the non-autonomous Julia set of the sequence $(f_{c_n})_{n=1}^{\infty}$ is Hölder continuous, provided $(c_n)_{n=1}^{\infty} \subset \overline{D}\left(0, \frac{1}{4}\right)$.

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Evaluation of Lebesgue Constant with Admissible Meshes over Balls and Simplices

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key-words: Lebesgue constant, admissible meshes.

Abstract:
We will present applications of admissible meshes for evaluating the Lebesgue constant and associated approximation errors for certain classes of polynomial approximation projections, including polynomial interpolation and weighted least squares. We will construct certain optimal admissible meshes over balls and simplices, derived from Chebyshev-type point sets, and give numerical comparisons with other known admissible meshes studied in recent years. The numerical experiments will be based on the covering radius with respect to the Dubiner distance. We will also discuss potential improvements to these constructions and open problems in this area.

The talk will be based on two papers [1, 2].

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Exploring State-Measure Dynamics in Algebraic Logic Structures

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key-words: State-Measure, algebraic Logic, mathematical modelling.

Abstract:
In this study, we investigate measures within algebraic logic frameworks utilizing the Sheffer stroke operation. We introduce and analyze various types of measures, including state-measures, measure-morphisms, and state-measure-morphisms, establishing their foundational axioms. We develop systematic algorithms to rigorously assess whether a given mapping satisfies the criteria for these measures. Our exploration extends to key algebraic characteristics, such as their antitonic nature and their role in defining deductive systems. Furthermore, we examine the relationship between measures and state functions, like Rie an and Bosbach states, showing that measures can be derived from these states and vice versa. Extremal state-measures are linked with affine homeomorphisms, providing a geometric perspective on the structural attributes of these algebraic logical structures. Theoretical insights are enriched with illustrative examples and proofs, laying a foundation for future research in fields like fuzzy logic, probabilistic reasoning, and the mathematical modeling of uncertainty.

Additionally, integrating this work into practical applications could prove valuable in areas such as uncertainty, fuzzy logic, quantum theory, and deductive physical models. Sheffer stroke algebras and measure-related mathematical structures offer a flexible and abstract tool for modeling the physical world, providing a robust foundation for analyzing complex systems. In practical terms, the next step involves exploring how these measures can be applied concretely, particularly in fields dealing with uncertainty, such as quantum mechanics and statistical physics. These mathematical structures can more accurately represent uncertainty in the physical realm.
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Mappings Contracting Perimeter of Triangles in Perturbed Metric Spaces

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key-words: Fixed point, mappings contracting triangles, perturbed metric spaces.

Abstract:
In the present paper we introduce the notion of mappings contracting perimeter of triangles in perturbed metric spaces, which we call perturbed mappings contracting perimeters of triangles. We provide a fixed point result for such mappings. We illustrate that our results are more general with some examples.

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Robust numerical method for the singularly perturbed problem with a nonlocal condition

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key-words: Numerical Method, finite difference method, uniform mesh.

Abstract:
We study a useful numerical method for the singularly perturbed problem with a non-local condition. A small positive parameter is used to determine the coefficients of the terms containing the highest-order derivative in singular perturbation situations. A uniform mesh is used to build finite difference schemes. We examine these schemes' uniform convergence and stability in the discrete maximum norm with respect to the perturbation parameter. The provided numerical experiments validate the accuracy of the proposed method.

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An Approximation Study with Mellin Integral Operators

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key-words: Logarithmic moments, order of approximation, Voronovskaya-type theorem.

Abstract:
In this presentation, we give an overview of some results from the literature and introduce a modification of Mellin convolution-type operators. In this way, we obtain the rate of convergence with the modulus of the continuity of the m th-order Mellin derivative of function f, but without the derivative of the operator. We show moments of the new operators and describe the behavior of the modified operators. Then, we express the Taylor formula including Mellin derivatives with integral remainder. Later, a Voronovskaya-type theorem is proved. In the last part, we state order of approximation of the modified operators, and the obtained results are restated for the Mellin-Gauss-Weierstrass operator.

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Approximation Properties of Neural Network and Sampling Kantorovich Operators in Terms of Image Reconstruction

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key-words: Neural Network operators, sampling Kantorovich operators, digital image processing.

Abstract:
In this talk, we present a comparative study between two powerful image reconstruction algorithms: the multivariate Neural Network (NN) operators, recently implemented for image reconstruction in [1], and the well known sampling Kantorovich (SK) operators (see, e.g. [4]). NN operators have recently attracted considerable attention, and their approximation properties have been thoroughly studied (see, e.g. [3, 5]). On the other hand, SK operators, known for their effectiveness in approximating not necessarily continuous signals, have proven to be successful in various applied fields (see, e.g. [2, 6]). We implement and analyze both methods on a dataset of reference images, evaluating their performances using three widely recognized similarity metrics: SSIM, PSNR, and the recently introduced FLIP index. Our results highlight the strengths and potential applications of each approach in the context of digital image processing.

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New Generalization of Sampling Type Series and Applications to Image Processing

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key-words: Sampling type operators, weighted approximation, rate of convergence, image processing.

Abstract:
In this presentation, we introduce modified sampling-type operators that generalize the classical sampling operators by incorporating a transformation function ρ. The talk begins with essential definitions, including the introduction of the weighted modulus of continuity and its fundamental properties. The newly constructed operators are analyzed in terms of pointwise and uniform convergence within spaces of continuous functions. We explore the weighted approximation properties of this family of operators in weighted function spaces defined by ρ. Furthermore, we examine the modular convergence of these operators in Orlicz spaces $L^{\eta}(\mathbb{R}^2)$. Lastly, we present certain ρ-kernels and their applications in image processing, demonstrating that the newly constructed operators provide improved performance and achieve higher PSNR values for specific parameters w and function ρ.
Acknowledgements: The first author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The author thanks to TUBITAK for their supports.

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Geometric Evolution of Space Curves According to Positional Adapted Frame in Euclidean 3-Space

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key-words: Inextensible fow, positional adapted frame, Euclidean space.

Abstract:
In this study, the inextensible flows of a spatial curve obtained using a positional adapted frame in Euclidean 3-space. Then, surfaces constructed with the help of the positional adapted frame, and the flows of these surfaces through inextensible curves examined. Finally, to verify the theoretical findings, various examples provided and visualized using drawing software.

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Approximation of Continuous Functions by Goodman-Sharma Type Modifications of the Bernstein Operator and the Baskakov Operator

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key-words: Bernstein operator, Baskakov operator, Goodman-Sharma operator, direct theorem, strong converse theorem, K-functional.

Abstract:
We suggest Goodman-Sharma type modifications of the Bernstein operator for approximation of continuous functions on $[0, 1]$ and of the Baskakov operator for approximation of continuous functions on $[0, \infty)$. For the related differential operators and the approximation operators we obtained nice commutative properties in each of the cases. Also, we proved direct and strong converse theorems with respect to the associated K-functionals, see [2, 3]. Although the operators we consider are non-positive, they have the advantage of $O\big(n^{-2}\big)$ rate for the approximation error compared to $O\big(n^{-1}\big)$ of the classical operators.
Particular results concerning the topic was obtained by Acu and Agrawal [1] for the case of the Bernstein operator and by Finta [4], Jabbar and Hassan [5] for the Baskakov operator, etc.
Anyway our approach is much more general, covers both cases, and the results are stronger.
In addition, numerical results of the approximation by the Baskakov-Goodman-Sharma operator are given, as well as computational aspects are discussed.
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A Generalized Approach to the Definition of Baskakov-Durrmeyer Operators

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key-words: Baskakov-Durrmeyer operators, uniform convergence, weighted modulus of continuity.

Abstract:
In the current study, we introduce a novel class of Baskakov-Durrmeyer operators, expanding the existing framework of positive linear operators. We derive sufficient conditions under which these operators exhibit weighted uniform convergence on the semi-infinite interval $[0, \infty)$, and provide precise quantitative estimates within appropriately defined weighted function spaces. Additionally, by establishing a Voronovskaya-type asymptotic formula, we rigorously prove the pointwise convergence behavior of the operators, thereby gaining deeper insight into their approximation properties.

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Approximation Properties of Szász Mirakyan Durrmeyer Operators Preserving Exponential Functions

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key-words: Szász Mirakyan Durrmeyer operators, weighted modulus of continuity, uniform convergence.

Abstract:
This paper serves as a natural continuation of the work presented in [8], where the Szász-Mirakyan-Durrmeyer operators that preserve exponential functions were initially introduced. As a preliminary result, we demonstrate that the sequence of norms of these operators, when acting on weighted function spaces with varying exponential weights, remains uniformly bounded. Subsequently, we derive Korovkin-type approximation theorems grounded in exponential weighted convergence. Finally, we provide a detailed characterization of the uniform approximation errors of the operators with respect to the exponential weights.

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3rd International Conference: Constructive Mathematical Analysis

Development of a New Efficient Algorithm for Multiplication of Large Integers

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key-words: Integer multiplication, complexity, computer arithmetic.

Abstract:
In most of the public key cryptography algorithms it is essential to develop efficient long integer multiplications. To achieve efficient computations in this area, several techniques are developed such as Toom-Cook, Karatsuba, Harvey's method, FFT, etc. In this study, we aim to construct a new effective algorithm for evaluating nth power of big integers. We will use divide and conquer approach to deal with the complicated multiplication process. Our attempts to solve the problem will begin with considering numbers with two-digit and larger numbers with base 10, then we generalize our formula to big integers and all bases s, where $2 \leq s \leq 10$. We will prove our results and support our theoretical findings by MATLAB experiments. Taking into account the complexity degree of the algorithm, implementation of this work will be a useful application for computer science.
Acknowledgements: This research is supported by Istanbul Sabahattin Zaim University grant number 2024-BAP 400-019.

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On Some New Perspectives on Hardy-Amalgam Spaces with Variable Exponents

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key-words: Amalgam spaces, Hardy-amalgam spaces, variable exponents analysis.

Abstract:
In this work, we present a generalization of Hardy spaces with variable exponents. We manage the functions in these spaces using atoms. Moreover, we investigate some mapping properties of the fractional integral operator. As an application, the boundedness of some operators in Fourier analysis is studied.

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Commutator of Anisotropic Sharp Maximal Operator on Lorentz Spaces

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key-words: Anisotropic maximal operator, commutator, Lorentz space.

Abstract:
In this talk we consider the commutator of the anisotropic sharp maximal operator $[b, M^{d,\sharp}]$ on the Lorentz spaces $L^{p,q}(\mathbb{R}^n)$. We obtain necessary and sufficient conditions for the boundedness of the commutator of the anisotropic sharp maximal operator $[b, M^{d,\sharp}]$ on the Lorentz spaces $L^{p,q}(\mathbb{R}^n)$ when b belongs to the bounded space of mean oscillations $BMO(\mathbb{R}^n)$, thereby obtaining some new characterizations for some subclasses of $BMO(\mathbb{R}^n)$, see [1, 2].

For $f \in L^1_{\rm loc}(\mathbb{R}^n)$, the anisotropic maximal operator M^d and the commutator of the anisotropic sharp maximal operator $[b, M^{d,\sharp}]$ are defined by

$$M^{d,\sharp}f(x) = \sup_{r>0} |\mathcal{E}(x,r)|^{-1} \int_{\mathcal{E}(x,r)} |f(y) - f_{\mathcal{E}(x,r)}|dy, \ [b, M^{d,\sharp}]f(x) = b(x)M^{d,\sharp}f(x) - M^{d,\sharp}(bf)(x),$$

where $\mathcal{E}(x,r)$ is the ellipsoid of radius r centered at $x \in \mathbb{R}^n$ and $|\mathcal{E}(x,r)|$ denotes the Lebesgue measure of $\mathcal{E}(x,r)$. Suppose that f is a measurable function on $\mathbb{R}^n$, then we define

$$f^*(t) = \inf\{s > 0 : d_f(s) \le t\}, \quad d_f(s) := |\{x \in \mathbb{R}^n : |f(x)| > s\}|, \quad \forall s > 0.$$

The Lorentz space $L^{p,q} \equiv L^{p,q}(\mathbb{R}^n)$, $0 < p, q \le \infty$ is the collection of all measurable functions f on $\mathbb{R}^n$ such the quantity

$$\|f\|_{L^{p,q}} := \|t^{\frac{1}{p}-\frac{1}{q}} f^*(t)\|_{L^q(0,\infty)}$$

is finite. Clearly $L^{p,p} \equiv L^p$ and $L^{p,\text{ß}} \equiv WL^p$. The functional $\|\cdot\|_{L^{p,q}}$ is a norm if and only if either $1 \le q \le p$ or $p = q = \infty$.

Theorem 3 *Let $p, q \in (1, \infty)$. The following assertions are equivalent:*

(i) $b \in BMO(\mathbb{R}^n)$ *and* $b^- \in L^\infty(\mathbb{R}^n)$.

(ii) *The operator $[b, M^{d,\sharp}]$ is bounded on $L^{p,q}(\mathbb{R}^n)$.*

(iii) *There exist a constant $C > 0$ such that* $\sup_{\mathcal{E}} \frac{\left\| \left(b(\cdot) - 2M^{d,\sharp}(b)(\cdot)\right)\chi_{\mathcal{E}} \right\|_{L^{p,q}(\mathbb{R}^n)}}{\|\chi_{\mathcal{E}}\|_{L^{p,q}(\mathbb{R}^n)}} \le C.$

(iv) *There exist a constant $C > 0$ such that* $\sup_{\mathcal{E}} |\mathcal{E}|^{-1} \left\| \left(b(\cdot) - 2M^{d,\sharp}(b)(\cdot)\right)\chi_{\mathcal{E}} \right\|_{L^1(\mathbb{R}^n)} \le C.$

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Characteristically Near Groups in Complex Plane Vector Fields

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key-words: Characteristic, complex plane, distance, group, nearness, vector field.

Abstract:
Characteristically near groups arise from variants of the Stein-Weiss characteristic function $\varphi(x) = e^{\pm ix}, x \in \mathbb{R}$ from [3], which defines the characteristic of a multiplicative abelian group $(\{\varphi(x)\}, \cdot)$, i.e.,

$$\varphi : \Omega \in 2^{\mathbb{C}} \to \mathbb{C} \text{ defined by } \varphi(x) = e^{2ix} : (G, \cdot) = (\{\varphi(z)\}, \cdot).$$

Characteristic functions were introduced by W.R. Hamilton in 1837 in a study of light rays [1]. The characteristic vector $\vec{\Phi}(G) = (\varphi_1(G), \varphi_2(G), \ldots, \varphi_n(G))$ contains characteristics $\varphi_i(t \in \Omega) \in [0, 1]$, which reflect the structure G, e.g.,

$$\varphi_1(G) = k \ \in \{0 \text{ (not in unit circle in } \mathbb{C}), 1 \text{ (in unit circle in } \mathbb{C})\}.$$
$$\varphi_2(G) = k \ \in \{0 \text{ (not abelian), } 1\text{(abelian)}\}.$$
$$\varphi_3(G) = o(G) \in \mathbb{Z}^{0+} \text{ (order of } G).$$

A characteristic distance (denoted by $d^{\Phi} : 2^X \times 2^Y \to \mathbb{C}$ [2] is used to measure of the nearness of vector field groups, defined by

$$d^{\Phi}(A, B) = \inf_{\substack{a \in A \\ b \in B}} \{|\varphi(a) - \varphi(b)|\} = \varepsilon \in [0, 1], (\varphi(a), \varphi(b)) \in \Phi(A) \times \Phi(B).$$

Vector field groups $\vec{\Phi}(G_1), \vec{\Phi}(G_2)$ are near, provided $d^{\Phi}(\vec{\Phi}(G_1), \vec{\Phi}(G_2)) = 0$.

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Local Fractional Hilbert-Type Inequalities

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key-words: Hilbert inequality, conjugate parameters, homogeneous function, local fractional calculus.

Abstract:
An intriguing topic related to classical inequalities is their application to fractal spaces through local fractional calculus. The principal function of local fractional calculus is to address diverse non-differentiable issues arising in intricate systems of real-world phenomena. The non-differentiability observed in science and engineering has been represented by local fractional ordinary or partial differential equations. The local fractional calculus, while originating from real-world occurrences, is also a significant instrument in pure mathematics.

Recently, some classical real inequalities have been extended to apply within specific fractal environments. A comprehensive compilation of generalisations encompassed inequalities featuring more generalised kernels, weight functions, and integration domains, as well as extensions to a multidimensional context. Special attention is given to a category of inequalities featuring a homogenous kernel. Specifically, one establishes some weak circumstances under which the constants on the right-hand sides of these local fractional inequalities are optimal. Additionally, improved and inverted relations are derived in a general multidimensional setting.

The primary aim of this paper is to investigate new local fractional Hilbert-type inequalities. We use our general results on non-homogeneous kernels. The equivalent form and several specific cases are derived as applications.

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Convergence Types on Interpolative Metric Spaces

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key-words: Summability Methods, interpolative metric spaces, statistical convergence.

Abstract:
In recent years, interpolative metric spaces have been developed as a flexible expansion of classical metric spaces, supporting the study of convergence theory and fixed point theory in expanded situations [3]. This study investigates the relationship between convergence types approaches in the context of interpolative metric spaces [2], [4]. Additionally, we present illustrative examples and establish first results that may serve as a basis for further research in the field of convergence theory and fixed point theory [1]. This research contributes to a comprehensive approach aimed at unifying various convergence techniques into generalized metric frameworks.

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Extended Semi-Parallel Tensor Product Surfaces in $\mathbb{E}_2^4$

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key-words: Tensor product surface, Gaussian curvature, extended semi-parallel.

Abstract:
Tensor product immersion of two immersions of a given Riemannian manifold has begun to be studied by B.Y. Chen. In the light of Chen's definition, many researchers studied the tensor product of two immersions. Then, tensor product surfaces of Euclidean plane curves were investigated by Mihai and Rouxel. Moreover, tensor product surfaces of Lorentzian plane curves were investigated by Mihai et al. Recently Bulca and Arslan studied some special semi-parallel surfaces in Euclidean spaces. Further, Yildirim and Ilarslan considered tensor product surfaces in 4-dimensional semi-Euclidean space with index 2, $\mathbb{E}_2^4$, satisfying the semi-parallelity condition. In present study, we consider and characterize tensor product surfaces in $\mathbb{E}_2^4$ satisfying the extended semi-parallelity condition.
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Power Series Statistical Relative Uniform Convergence of a Double Sequence of Functions at a Point and Approximation Results

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key-words: Double sequence, Korovkin theorem, power series statistical convergence.

Abstract:
In this presentation, we introduce a novel concept in statistical convergence for double sequences, named power series statistical uniform convergence at a point, defined with respect to the power series method. We establish an approximation theorem for sequences of functions under this convergence. Furthermore, we provide an illustrative example that satisfies our new theorem but fails to hold under previously studied convergence methods. Additionally, we investigate the rate of convergence, offering a quantitative analysis of the proposed method's efficiency.

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On the Convergence of Generalized Szász-Mirakjan Operators in the Variation Seminorm

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key-words: Linear positive operators, generalized Szász-Mirakyan operators, variation seminorm, variation detracting property, rate of convergence, modulus of continuity, convergence in variation seminorm, $TV[I]-$space, Voronovskaya-type theorem.

Abstract:
The aim of the present paper is to introduce generalized Szász-Mirakyan operators. This article studies the property of variation seminorm and on some approximation properties of generalized Szász-Mirakyan operators not only in normed spaces but also in variation seminorm. We obtain convergence properties of our operators with the help of Korovkin's theorem and the order of convergence by using a classical approach, the second modulus of continuity and Peetre's K-functional. We also give asymptotic formula and the convergence of the derivatives for these operators. We investigate the variation detracting property of generalized Szász-Mirakyan operators. We show the convergence of generalized Szász-Mirakyan operators in variation seminorm and its rate of convergence in variation seminorm.

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A Modification of Bernstein Operators on the Interval $(-1, 1)$

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key-words: Bernstein operators, modulus of continuity, rate of convergence.

Abstract:
In this talk, we will introduce a modification of Bernstein operators, which are defined on the interval $(-1, 1)$ and we will also discuss the approximation properties of these operators, including the rate of convergence, weighted approximation and asymptotic formulas. Lastly, the approximation will be supported with graphs.

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On Approximation by Algebraic Version of the Trigonometric Jackson Integrals $G_{s,n}$ in Generalized Weighted Integral Metric

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key-words: Linear operator, direct theorem, strong converse theorem, K-functional.

Abstract:
We characterize the errors of approximation of the algebraic version of trigonometric Jackson integrals $G_{s,n}$ in weighted integral metric with weight of the form $(1-x)^{-\frac{1}{2}-\gamma_1}(1+x)^{-\frac{1}{2}-\gamma_2}$ for $\gamma_1, \gamma_2 \in [0, 1/2)$. We prove direct and strong converse theorem in terms of the weighted K-functional. The results extend the case $\gamma_1 = \gamma_2 = 0$ to weights with negative powers.

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Estimating Covariance in Generalized Least Squares for Frechet Distribution

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key-words: Generalized least squares, weighted least squares, covariance matrices.

Abstract:
The Frechet (extreme value type **II**) distribution is one of the probability distributions used to model extreme events. We consider two-parameter Frechet distribution with shape parameter α, scale parameter β.

After some algebraic manipulation and for a sample size of n and $x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n)}$ the regression model is rewritten as:

$$\ln x_{(i)} = -\ln \beta - \frac{1}{\alpha} \ln \left(-\ln \left(F(x_{(i)})\right)\right) \tag{1}$$

If we replace $\ln x_{(i)}$ with Y_i , $-\ln \beta$ with a_0, $-\frac{1}{\alpha}$ with a_1 and $\ln \left(-\ln \left(F(x_{(i)})\right)\right)$ with X_i ,the regression model (1) occurs as:

$$Y_i = a_0 + a_1 X_i \tag{2}$$

In the linear regression model given in (2), if the form of the variance of $Y = (Y_1, ..., Y_n)$ is $\sigma^2 \mathbf{V} = \Sigma$, GLS minimizes

$$\hat{\theta}_{GLS} = (X^t V X)^{-1} XVY$$

where $\hat{\theta}_{GLS}$ is the vector of the GLS estimates of $\hat{\theta} = (a_0, a_1)$ and X is the matrix of ones and x_i.

Now, the problem is to estimate the $\mathbf{V}$ matrix for the considered distributions.by using Taylor expansion [2], it is possible to approximate the expectation of the observation and also variance and covariance between the observations.

the approximate formula for the $\mathbf{V}$ matrix can be expressed as follows:

$$v_{ij} = \frac{i}{(n+1-i)} \frac{1}{\frac{i}{n+1}(-\ln(\frac{i}{n+1}))} \frac{1}{\frac{j}{n+1}(-\ln(\frac{j}{n+1}))}, \ \ i \prec j$$

where v_{ij} is an element of the $\mathbf{V}$ matrix.

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3rd International Conference: Constructive Mathematical Analysis

Numerical Study and Convergence of a Finite Element Method for the Kazhikhov-Smagulov-Korteweg Pollution System

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key-words: Numerical simulation, Kazhikhov-Smagulov-Korteweg equations, viscous incompressible fluid mixture.

Abstract:
The numerical modeling of pollutant transport in mixtures of viscous incompressible fluids plays a crucial role in environmental applications, particularly in wastewater treatment and pollution control. In this study, we conduct a detailed numerical analysis of a Kazhikhov-Smagulov-Korteweg-type model, which describes the behavior of such fluid mixtures with capillary effects and variable density. We first establish a variational formulation of the system that accounts for incompressibility, viscosity, capillarity, and source terms related to pollutant transport. Based on this formulation, we construct a fully discrete scheme using finite element methods for spatial discretization and a second-order Backward Differentiation Formula (BDF2) for time integration, applied to both the velocity and density variables. The proposed scheme is proven to be unconditionally stable using the G-norm framework specific to BDF2 methods. We also prove the convergence of the numerical solution to a global weak solution of the original system. To illustrate the relevance of the model, we simulate a practical scenario involving the dispersion of a chemical pollutant in treated wastewater, where differences in viscosity and local density gradients play a significant role. The results confirm the ability of the method to capture realistic pollutant dynamics and demonstrate its robustness in simulating complex environmental flows.

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On a Two-layer Conjugation Problem for the Heat-Conduction Equation in a Cylinder

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key-words: Two-layer medium, temperature field, unsteady heat conduction, integral transform method, conjugation conditions.

Abstract:
The analytical theory of unsteady heat conduction in a cylindrical surface finds wide application in solving important technical and technological problems of heat- and mass-transfer processes that express energy or material balances. This paper examines an initial-boundary-value problem for the heat-conduction equation on a cylindrical surface with a prescribed initial temperature distribution, boundary conditions, and conjugation conditions on the contact cylindrical surface. The conjugation boundary condition (of the fourth kind) corresponds either to heat exchange between a body's surface and the surrounding medium according to Fourier's law of conduction or to heat exchange between contacting solids when the temperatures of the contacting surfaces are identical (ideal, perfect thermal contact). In addition, when a solid body is washed by a fluid (or gas), the heat transfer from the fluid (gas) to the body's surface in the immediate vicinity of that surface occurs by molecular heat conduction, which likewise leads to heat exchange described by the fourth-kind boundary condition.According to the second law of thermodynamics, a spontaneous heat-transfer process in space arises when there is a temperature difference and proceeds in the direction of decreasing temperature. Thermodynamic processes are accompanied by heat exchange between structural elements within the working space of thermal installations and the surrounding environment, where heat is a quantitative measure of energy; the regularities of these heat-transfer processes are treated by the theory of heat exchange and heat transfer.Using the method of separation of variables together with integral transforms—the Laplace transform with respect to time and the zero-order Bessel transform with respect to the radial coordinate—a closed-form analytical solution to the unsteady two-layer heat-conduction problem is obtained. The investigation of initial-boundary-value problems for the heat-conduction equation and the development of analytical solution methods constitute an important and rapidly evolving area of mathematical physics. Solutions of this type can be applied to compute transient temperature fields and heat fluxes in two-layer sheet products, structures, and buildings, as well as in cylindrical specimens and apparatuses whose thermophysical parameters depend functionally on temperature and on interfacial boundaries. They can also be used to determine the conditions under which two-layer heat-exchange processes proceed, based on temperature fields and the corresponding initial and boundary conditions. The practical significance of the results lies in solving concrete initial-boundary-value problems of thermophysical processes.

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A Numerical Approach Based on a Dimensionality Reduction Procedure for Approximating Multiple Integrals

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key-words: Multiple integral, numerical integration, α-dense curves.

Abstract:
This paper explores a dimensionality reduction procedure using parametric α-dense curves with a minimal density α for approximating multiple integrals. By utilizing specific α-dense curves $\ell_\alpha(t)$ of finite length, the multiple integral of a positive continuous function f over Ω can be approximated by a single integral that corresponds to the length of $\ell_\alpha(t)$ and the density α. Finally, we provide some numerical examples.

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